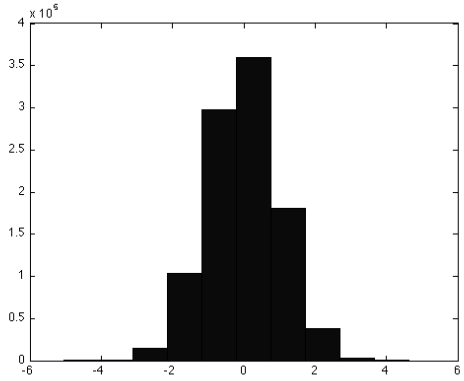
Density Curves and the Normal Distributions

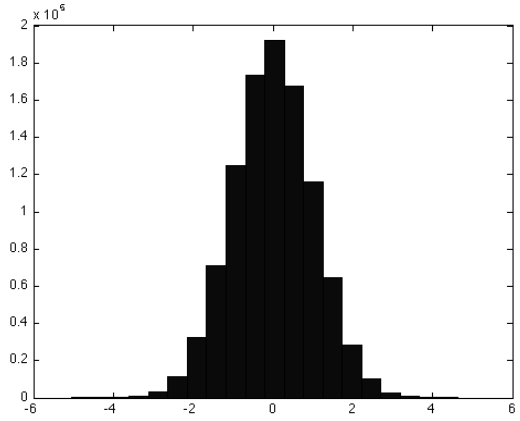
MATH 2300

Chapter 6

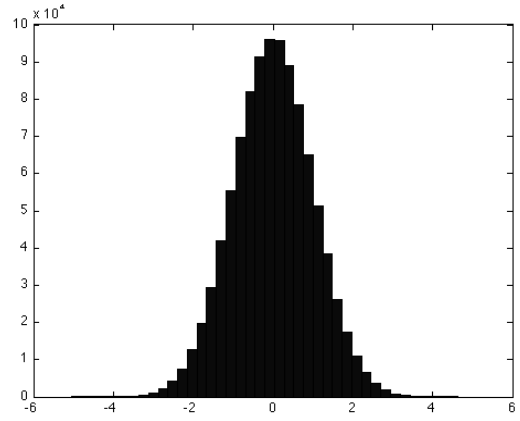
Histogram: 10 groups



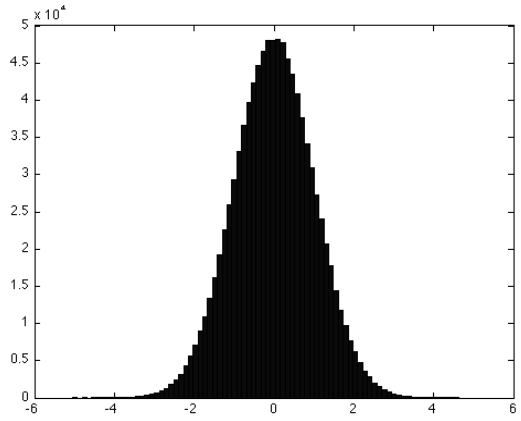
Histogram: 20 groups



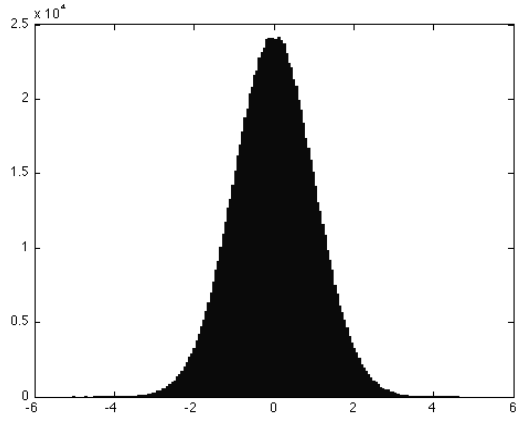
Histogram: 40 groups



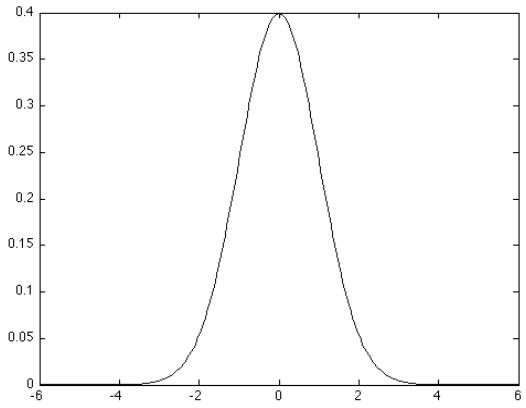
Histogram: 80 groups



Histogram: 160 groups



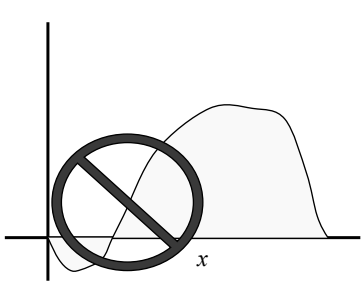
Density Curve

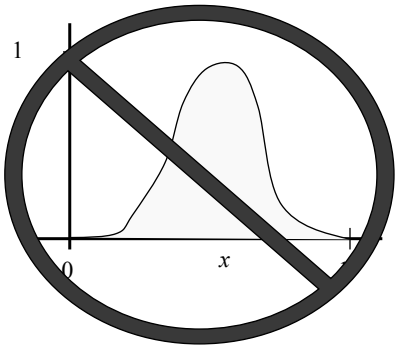


Density Curves

- Describe the overall pattern of a distribution
- Always on or above the horizontal axis
- Total area underneath the curve = 1
- Used to find proportion of data within a given range of values.

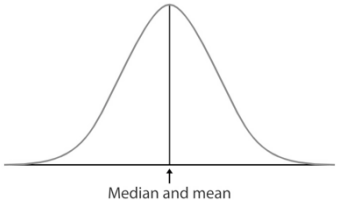
Are these density curves?



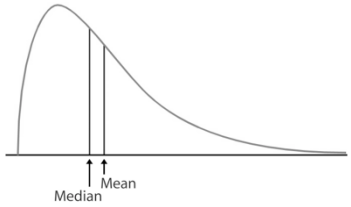


Mean and Median in a Density Curve

Symmetric

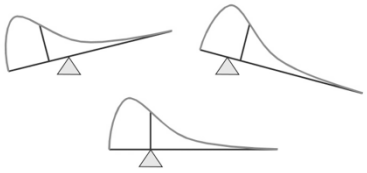


Skewed



The median divides the curve into equal areas.

The mean is the balance point of the curve.



Density Curve Notation

- We distinguish between the mean and standard deviation of the observations and those of density curves.
- For density curves:

  Mean = μ

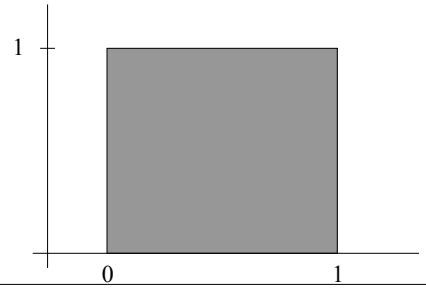
  Standard Deviation = σ

Two Important Families of Densities

- Uniform Distributions
- Normal Distributions

Uniform Distributions

- All data values have the same density.
- The proportion of data in a given range is calculated as: $range \times height$

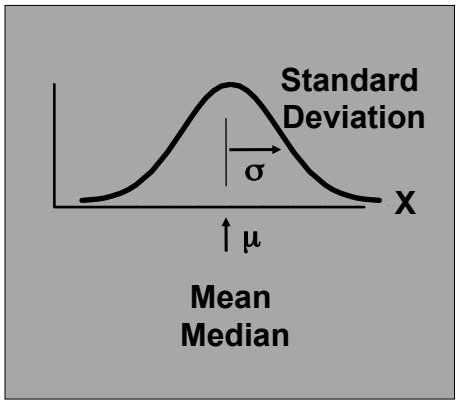


Example: Reaction Temperature

- The reaction temperature x (in °C) for a certain chemical reaction has a uniform distribution between-5 and 5.
- Compute the proportion of temperatures:
  - Less than 0
  - Between -2.5 and 2.5
  - Between -2 and 3

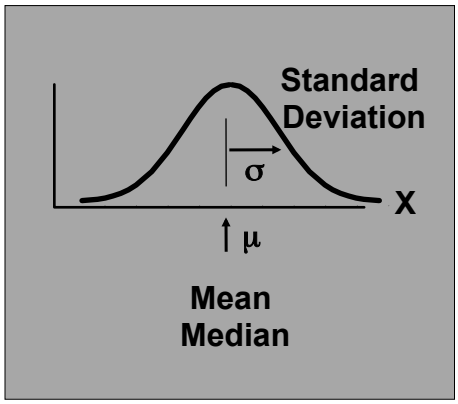
Normal Distributions

- Bell shaped
- Symmetric
- Mean and median of the curve are equal

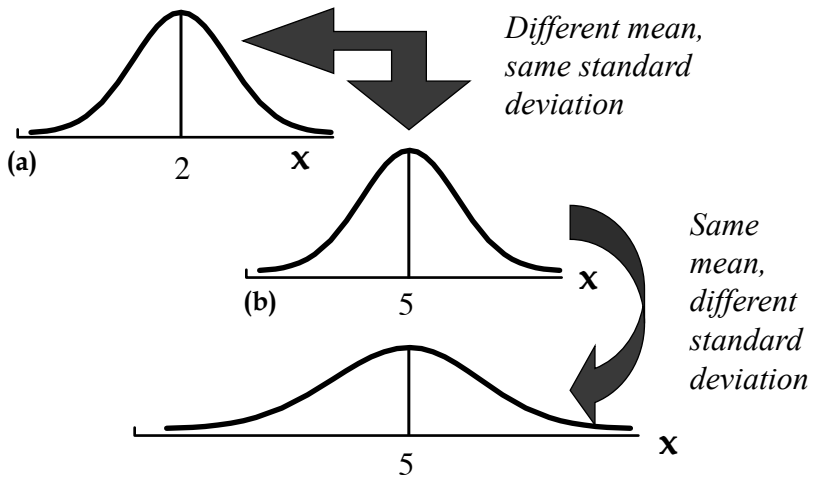


Normal Distributions

- Data values range from -∞ to +∞
- Completely described using only μ and σ
- Denoted by: $N(\mu,\sigma^2)$



Differences Between Normal Distributions



Different mean, same standard deviation

Same mean, different standard deviation

The 68-95-99.7 Rule (Empirical Rule)

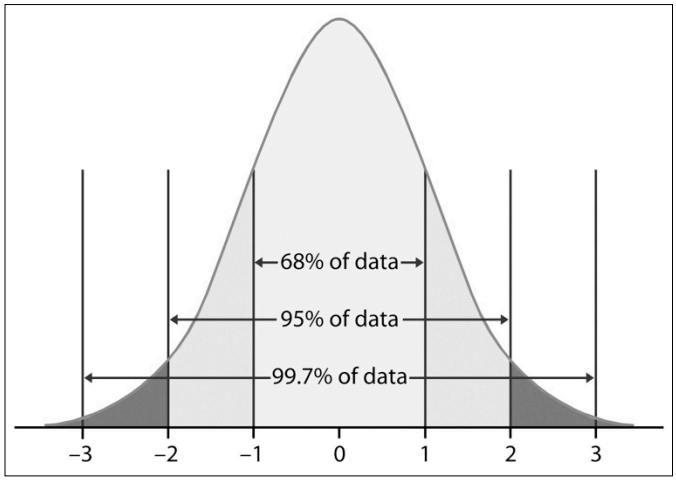
THE 68–95–99.7 RULE

In the Normal distribution with mean μ and standard deviation σ:

- 68% of the observations fall within σ of the mean μ.
- 95% of the observations fall within 2σ of μ.
- 99.7% of the observations fall within 3σ of μ.

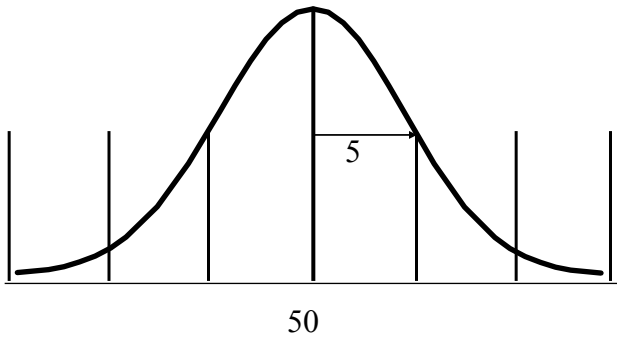
NOTE: These are just approximations

The 68-95-99.7 Rule (Empirical Rule)



Example

Scores on a Law test for a first-year class are approximately normal with μ=50 and σ=5.

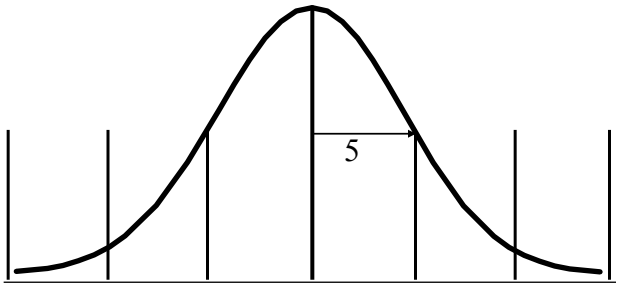


Example

About what percent scored below 50?

In what range do the middle 68% lie?

About what percent scored above 65?



The Standard Normal Distribution

- The *standard normal distribution* is a normal distribution where
- The mean = 0.0
- The standard deviation = 1.0
- Measures the number of standard deviations a point is from the mean.
- Positive z-values are above the mean and negative z-values are below.

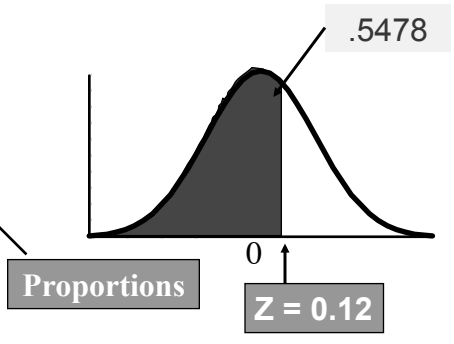
Finding Normal Proportions

- There is no formula for calculating these.
- To obtain them, either software or ***tables*** are used.
- Table II provides **cumulative percentages**
  - The cumulative proportion for a value *x* is the proportion of observations in the distribution that are **less than or equal to** *x*.

Using Table II

Partial Cumulative Standard Normal Distribution Table

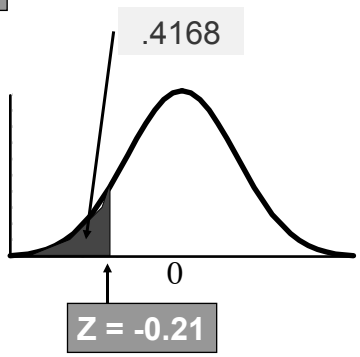
Z	.00	.01	**.02**
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255



Using Table II

Partial Cumulative Standard Normal Distribution Table

Z	.00	.01	**.02**
-.30	.3821	.3783	.3745
-.20	.4207	.4168	.4129
-.10	.4602	.4562	.4522
0.0	.5000	.4960	.4920



Example

Find the proportions corresponding to the following statements:

a) $z < 2.85$

b) $z > 2.85$

c) $z > -1.66$

d) $-1.66 < z < 2.85$

Standardizing

- If a variable *x* is from N(μ,σ^2), then the standardized value of x, called a **z-score** is the following:

$$z = \frac{x - \mu}{\sigma}$$

- The variable *z* is from N(0,1)

Example

Weights of baby elephants follow a Normal distribution with mean μ=224 lbs and standard deviation σ=53 lbs.

a) What percent are lighter than 200 lbs?

b) What percent are between than 200 and 300 lbs?

c) What percent are heavier than 445 lbs?

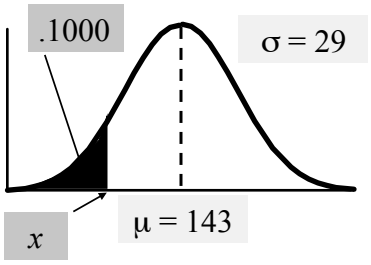
Finding a Value Given the Proportion

1. State the problem and draw a picture.
   - What are the mean and standard deviation? What proportion is given?
2. Use Table II.
   - Look for the entry closest to the given proportion to find the z-score.
3. Unstandardize the z-score.
   - Transform *z* back to the original *x* scale.

Example

The steel reinforcement bars manufactured in a foundry have lengths that follow a normal distribution with mean $\mu = 143$ in. and standard deviation $\sigma = 29$ in.

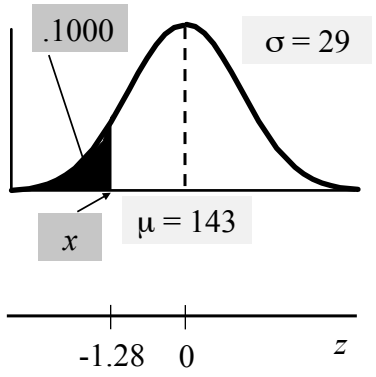
Below what length would 10% of the bars fall?



Example

- **Use Table II to find the z score corresponding to the region bounded by *x*.**

Z	.07	**.08**	0.9
-1.5	.0582	.0571	.0559
-1.4	.0708	.0694	.0681
-1.3	.0853	.0838	.0823
-1.2	.1020	**.1003**	.0985



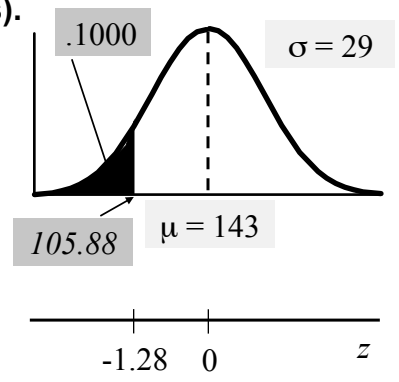
Example

- **Use the following formula to translate the *z* score into *x* (in the original units).**

$$x = \mu + z\sigma$$

$$x = 143 + (-1.28)\ 29$$

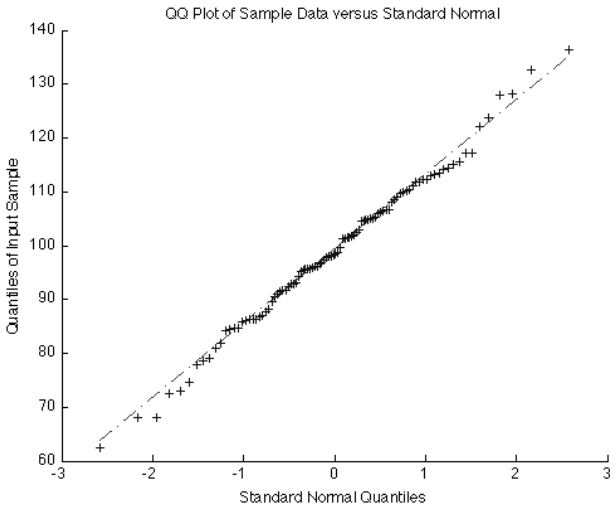
$$x = 105.88 \text{ in.}$$



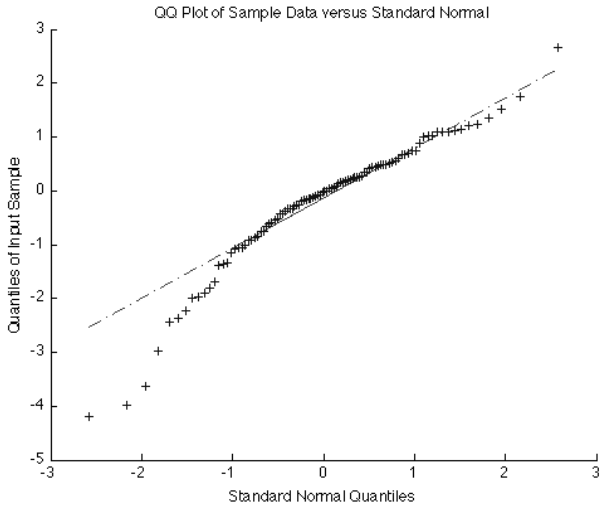
Normal Probability Plots

- **Plots sample percentiles against percentiles of the standard Normal distribution**
- **Provides method for determining whether an assumption of Normality is plausible:**
- **If the sample comes from $N(\mu,\sigma^2)$, the points should fall close to a line with slope σ and intercept μ.**

Example



Example



Example