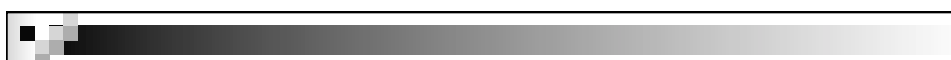


Measures of Center

MATH 2300
Section 3.1



Location and Variability

- To get a *useful* numerical description of a distribution, we need to have both some measure of **center (or location)** and some measure of **variability**.

Measures of Center

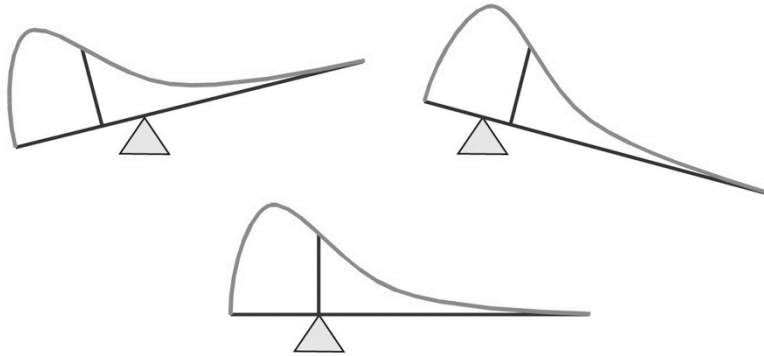
- Provide some notion of what value the distribution is grouped around.
- The most common measures are the **mean**, **median**, and **mode**.

The Sample Mean

- Also called the arithmetic **average**.
- Represents the *balance point* for a sample of *quantitative* data.

$$\bar{x} = \sum_{i=1}^n x_i = \frac{x_1 + x_2 + \dots + x_n}{n}$$

The Mean as Balance Point



Housing Price Example

All Values Given in Thousands of Dollars

{House Prices} = {144; 98; 204; 177; 155; 316; 100}

$$\begin{aligned}\bar{x} &= \frac{144 + 98 + 204 + 177 + 155 + 316 + 100}{7} \\ &= \frac{1,194}{7} = 170.6\end{aligned}$$



The Sample Median

- The median Q_2 is the **midpoint** of a distribution.
- It is the number such that half the observations are smaller and the other half are larger.



Calculating the Median

1. Arrange the observations in order from smallest to largest.
2. If n is odd, Q_2 is the center observation.
3. If n is even, Q_2 is the mean of the two centered observations.

Housing Price Example

{house prices} = {\$144; 98; 204; 177; 155; 316; 100}

NOTE: n is odd

Ordered:

\$98; 100; 144; 155; 177; 204; 316



Middle Value

$$Q_2 = 155$$

Another Housing Price Example

{house prices} = {144; 98; 204; 177; 155; 316; 100; 177; 177; 170}

NOTE: n is even

Ordered:

\$98; 100; 144; 155; 170; 177; 177; 177; 204; 316



Middle Values

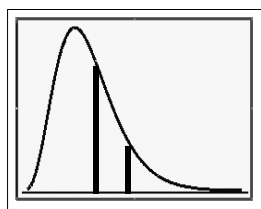
$$Q_2 = (170 + 177)/2$$

$$= 173.5$$

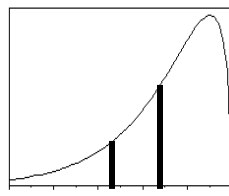
Mean vs. Median

- The mean is GREATLY affected by outliers.
- The median is NOT affected by outliers.
- If the mean and median are (almost) equal, then the distribution is (approximately) symmetric.
- If $\text{mean} < \text{median}$, distribution is left skewed.
- If $\text{mean} > \text{median}$, distribution is right skewed.

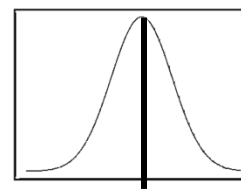
Skewness



Right-Skewed



Left-Skewed



Symmetric

Trimmed Means

- Remove a percentage of the highest and lowest values and calculate the mean from what remains.
- More **resistant** to outliers than the mean.
- Less **resistant** to outliers than the median.

Example

{house prices} = {\$144; 98; 204; 177; 155; 316; 100}

Mean = 170.6

Median = 155

Ordered:

~~\$98~~; 100; 144; 155; 177; 204; ~~316~~

14.3% Trimmed Mean = 156



The Mode

- The observation with greatest frequency in the data set
 - If no value occurs more than once, then the data set has ***no*** mode.
- Not useful for really small data sets
- Sometimes, it only makes sense to describe as a class, rather than an individual value