Additional Examples of the Central Limit Theorem in Practice



Example 1: Population Distribution

The distribution is heavily positively skewed



Example 1: Distribution of Sample Mean with n=15



Our rule of thumb does not hold here. The sampling distribution is still positively skewed. We will need an even large sample size for the CLT to work.



Even a sample size of 50 is still not sufficient here. We will need an even large sampler size for the CLT to apply.



Even a sample size of 100 is still not sufficient here. We will still need an even larger sample size for the CLT to apply.



This distribution is still not *quite* normally distributed with 200 observations, but it is much closer than what we saw with the smaller sample sizes. It wouldn't be a terrible idea to use the CLT with this sample size for this distribution.



Example 2: Population Distribution

The population has a bimodal distribution with peaks of different heights.



Even though the distribution is not quite normal yet, we see that it is already unimodal and bell-shaped with a slight positive skew, despite the population having two peaks.



With just 30 observations, we see that the sampling distribution already looks pretty close to being normally distributed. It's helpful to see what happens with even larger sample sizes, though.



The sampling distribution is even more normally distributed now than it was before! We also can see that the amount of variability has decreased, too, showing the Law of Large Numbers in action, as well.



The sampling distribution looks very much like a normal distribution now, so we should have absolutely no doubts about applying the CLT. We see again that the amount of variability has decreased even more, as well.



Example3: Population Distribution

This population is trimodal with peaks of different heights.



Once again, the sampling distribution is unimodal for a small sample size despite the population having many peaks.



Unlike the previous example, here 30 observations is not quite enough to get a sampling distribution that looks normal. It is still a little positively skewed.



The sampling distribution is closer to being normal than it was for 30 observations, but it is still slightly skewed positively. We can take a look at the probability plot of this distribution to help us understand it even better, though.



From this probability plot, we can see that, since it closely follows a linear pattern for most of the data, it is very close to being normal. The tails deviating from the line quite as much as they do, though, suggests that even more observations would be help improve the normality of the distribution.



The shape of the distribution here is still nearly identical to what we had for 50 observations, though the amount of variability has decreased considerably still. This shows that even though we might get close to having a normal distribution with a relatively small sample size, it might take many more observations to get a really good approximation.