SUPERSTABLE AND CRITICAL CONFIGURATIONS IN GENERALIZED CHIP-FIRING MODELS

SUHO OH

ABSTRACT. Chip-firing is a game played on a connected graph G, where chips are distributed across the vertices and move to adjacent vertices according to a simple rule. This dynamical system has deep connections to algebra, combinatorics, and physics. A well-known result in the classical chip-firing setting is the duality between superstable and critical configurations. Guzmán and Klivans introduced a natural extension of this system to (L, M) pairs, where L is an arbitrary invertible matrix and M generalizes the graph Laplacian. We present a new tool that simplifies the computation of superstable and critical configurations in this generalized model. Furthermore, we extend the classical duality to this broader framework. Additionally, we explore intriguing group-theoretic questions that emerge from this setting. This work is partially joint with Matthew Cho, Anton Dochtermann, Ryota Inagaki, Dylan Snustad, and Bailee Zacovic, and partially joint with Zach Benton, Jane Kwak, Mateo Torres, and McKinley Xie.