A GEOMETRIC ANALYSIS OF KOSZUL STRUCTURES FOR TRIVARIATE MONOMIAL IDEALS

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Abstract. The Koszul Algebra structure for codepth 3 commutative local rings has been studied significantly in recent years. Codepth 3 local rings can be described with 5 distinct classifications; $\mathbf{C}(c)$, $\mathbf{T}$, $\mathbf{B}$, $\mathbf{G}(r)$, and $\mathbf{H}(p,q)$, with $\mathbf{H}(p,q)$ being the most diverse of these classes. We describe geometric approaches to identify the Koszul structure of R/I when I is a monomial ideal in $R=\Bbbk[x,y,z]$ where $\Bbbk$ is a field. Specifically, if I is $\mathfrak{m}-$primary and $I\subseteq\mathfrak{m}^2$, where $\mathfrak{m}$ is the homogeneous maximal ideal of R, then the Koszul structure for R/I can be determined by analysis of the planar graph corresponding to the minimal free resolution of R/I. This allows us to bound the invariants, p and q, of the Koszul structure in terms of number of minimal generators, m, of I. Additionally, the type, n, of R/I corresponds to the number bounded faces of the planar graph, and is thus subject to Euler's bound, $n\leq 2m-5$. We will argue that if R/I has maximum type of $2m-5$, then R/I is of the $\mathbf{T}$ structure.