# SYZYGIES OF DETERMINANTAL THICKENINGS AND gl($m|n$) REPRESENTATIONS

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ABSTRACT. The coordinate ring $S = \mathbb{C}[x_{i,j}]$ of space of $m \times n$ matrices carries an action of the group $\mathrm{GL}_m \times \mathrm{GL}_n$ via row and column operations on the matrix entries. If we consider any $\mathrm{GL}_m \times \mathrm{GL}_n$-invariant ideal $I$ in $S$, the syzygy modules $\mathrm{Tor}_i(I, \mathbb{C})$ will carry a natural action of $\mathrm{GL}_m \times \mathrm{GL}_n$. Via BGG correspondence, they also carry an action of $\bigwedge^\bullet (\mathbb{C}^m \otimes \mathbb{C}^n)$. It is a result by Raicu and Weyman that we can combine these actions together and make them modules over the general linear Lie superalgebra $\mathfrak{gl}(m|n)$. We will explain how this works and how it enables us to compute all Betti numbers of any $\mathrm{GL}_m \times \mathrm{GL}_n$-invariant ideal $I$.