DEFINING IDEALS OF REES RINGS OF MAXIMAL MINORS OF SPARSE MATRICES

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Abstract. The study of rings and more generally of varieties that are defined by determinantal ideals of generic matrices has been a central topic in commutative algebra and algebraic geometry. We consider the ideals generated by maximal minors of sparse matrices of size 2 by n. Using the theory of Sagbi bases we describe the defining equations of the Rees algebra of such ideals. The Rees algebra and the special fiber ring of an ideal arise in the process of blowing up a variety along a subvariety. Rees rings and special fiber rings also describe, respectively, the graphs and the images of rational maps between projective spaces. A difficult open problem in commutative algebra, algebraic geometry, elimination theory, and geometric modeling is to determine explicitly the equations defining graphs and images of rational maps, and therefore describing the defining ideals of Rees rings. To understand the image of a map it is often better to study the graph of the map since it encodes more information than the image itself. Indeed, the defining ideal of the special fiber is immediately attained from the defining ideal of the Rees ring. The converse is usually not true, however if this the case, the ideal is called of fiber type. For maximal minors of 2 by n sparse matrices we show that I is of fiber type and the special fiber ring is defined by the Plücker relations. We obtain many consequences: the Rees algebra has rational singularities if the field has characteristic zero and is F-rational if the field is of positive characteristic. In particular, the Rees algebra is a Cohen-Macaulay normal domain. In addition, the Plücker relations together with the linear relations form a Groebner basis for the defining ideal of the Rees ring, hence the latter is Koszul and the ideal has linear powers. This is joint work with Ela Celikbas, Emilie Dufresne, Louiza Fouli, Elisa Gorla, Kuei-Nuan Lin, and Irena Swanson.