FULL ALGEBRAS OF MATRICES AND TRANSITIVE SYSTEMS

PIOTR WOJCIECHOWSKI

ABSTRACT. A subalgebra $\mathcal{A}$ of $M_n(\mathbb{R})$ is called *full* if for all $i = 1, \ldots, n$, $E_{ii} \in \mathcal{A}$. All such algebras are permutation-isomorphic to block lower-triangular matrices with corresponding subdiagonal blocks being either zero-blocks or full. Two full algebras are isomorphic if and only if they are permutation-isomorphic. A one-to-one correspondence is provided between the full algebras and transitive directed graphs. It is also proven that such algebras, if endowed with a lattice order, can be almost-f- or d-algebras only if they are diagonal. Moreover, for every totally ordered field $\mathbb{F}$, if all $E_{ii} > 0$ in a lattice-ordered full subalgebra $\mathcal{A}$ of $M_n(\mathbb{F})$, then $\mathcal{A}$ is isomorphic to the algebra ordered in the usual way.

By a *transitive system* we understand a tuple $(X, \mathfrak{R}, f, G)$ where X is a nonempty set, $\mathfrak{R}$ is a reflexive and transitive relation on X, G is a group and f is a *transitive function* defined on $\mathfrak{R}$ into G. This concept emerged naturally in determining the isomorphisms of full algebras of matrices. Some examples and properties of the transitive systems will be presented, together with their connection with the graph theory.