MULTIGRADED ALGEBRA OVER POLYNOMIAL RINGS WITH REAL EXPONENTS

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Abstract. Commutative algebra over polynomial rings with real exponents has become important in the past decade because of applications in Topological Data Analysis. Specifically, persistent homology with multiple parameters results in nothing more or less than multigraded modules over real-exponent polynomial rings. This talk explains what that means (in detail, from scratch) and covers the fundamentals of (computational) commutative algebra in this setting: presentation, primary decomposition, syzygy theorem, and so on. One of the major concerns is what to do about the uncountably spectacular failure of noetherian hypotheses, especially given the computational motivations for thinking about real exponents in the first place.

If you're interested, here are some warm-up exercises. In the two-variable real-exponent polynomial ring, find an ideal that is minimally generated by uncountably many monomials. Instruct a computer how to store your ideal. Compute a resolution of your ideal. Does your ideal have a primary decomposition? Show that the ideal of all real-exponent polynomials with constant term 0 is maximal and generated by countably many monomials, although it does not admit a minimal generating set. Do the same for the ideal of all real-exponent polynomials with no pure powers of (say) the first variable, but with "prime" instead of "maximal". It is not at all necessary to do these exercises to understand the talk, but they'll help you appreciate what's going on.