

COMBINATORIAL PROPERTIES OF BOREL IDEALS

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ABSTRACT. Borel ideals are monomial ideals whose monomials are closed under Borel moves, which is when a variable is replaced by one of a lower index (that is, x_j can be replaced with x_i whenever $i < j$). As one might guess from their definition, these ideals have several nice features, both algebraic and combinatorial. For example, given a set X of monomials, one can define $\text{Borel}(X)$ to be the smallest Borel ideal containing the set X . In this way, we can define the Borel generators of a Borel ideal. One can also weaken the Borel quality, requiring that x_j can be replaced with x_i only when i precedes j in some fixed partially ordered set. We discuss some background on Borel ideals, as well as original research along the lines discussed above. No knowledge or Borel ideals will be assumed. The original research is joint work with Chris Francisco and Jeff Mermin.