PRIMARY DECOMPOSITION OF BINOMIAL IDEALS

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Abstract. One of the driving problems in the development of number theory and commutative algebra was the search for generalized versions of the unique factorization of integers into prime powers. It was this problem that motivated the definition of an ideal, and it became important to find decompositions of ideals as intersection of primary ideals. In a landmark achievement, Emmy Noether elegantly used the ascending chain condition to show the existence of such decompositions, and rings where the a.c.c. holds are called noetherian in her honor precisely for this result.

Primary decomposition of polynomial ideals is particularly important, both in pure and applied mathematics. This motivated the development and implementation of algorithms for primary decomposition in polynomial rings over fields. Nevertheless, this remains a computationally expensive undertaking, and many examples arising in applications cannot be handled by general purpose procedures.

For special classes of ideals, where additional (combinatorial) structure is present, one may attempt to do primary decomposition "by hand". This is possible, indeed, easy, in the first combinatorial class of ideals everyone loves, that is, monomial ideals.

The study of the primary decomposition of binomial ideals (ideals generated by polynomials with at most two terms) was initiated by Eisenbud and Sturmfels, who showed that the associated primes and primary components of a binomial ideal (in a polynomial ring over an algebraically closed field) are binomial. These results, which are accompanied by specialized binomial primary decomposition algorithms, have proved to be very influential and useful. However, they are not at all explicit when it comes to describing the primary components themselves.

Recent work, both in the computational and combinatorial directions, now provides a satisfactory understanding of the primary components of a binomial ideal. The goal of this talk is to explain some of these results; I will include joint work with Dickenstein and Miller, and with Eser.