When is a closure operation both a Nakayama closure and a semiprime operation?

Janet C. Vassilev

University of New Mexico

Abstract: Let $(R, \mathfrak{m})$ be a local ring. We say $I \to I^c$ is a closure operation on the set of ideals of R if for all ideals $I \subseteq R$

(C1) $I \subseteq I^c$,
(C2) $I \subseteq J \Rightarrow I^c \subseteq J^c$ and
(C3) $(I^c)^c = I^c$.

We say a closure operation is a Nakayama closure if $I \subseteq J \subseteq (I + \mathfrak{m}J)^c$, implies that $I^c = J^c$. A closure operation is semiprime if $I^c J^c \subseteq (IJ)^c$. Several well known closure operations, such as integral closure and tight closure, are both Nakayama closures and semiprime operations. We will exhibit that there are Nakayama closures which are not semiprime and semiprime operations which are not Nakayama and discuss some conditions which will ensure a closure operation is both semiprime and Nakayama.