MATH 9099-001 HOMEWORK DUE 12 MARCH

(1) Exercise 4.4.6

(2) Consider ideal

$$I = \langle x^2 + y^2 + z^2 - 4, x^2 + 2y^2 - 5, xz - 1 \rangle \subset k[x, y, z]$$

   (a) Find the elimination ideals $_1I$ and $_2I$.
   (b) Find all solutions to the system

$$x^2 + y^2 + z^2 = 4$$
$$x^2 + 2y^2 = 5$$
$$xz = 1$$

(3) Consider the system of equations

$$x^2 + 2y^2 = 3$$
$$x^2 + xy + y^2 = 3$$

   (a) Let I be the corresponding ideal in $k[x, y]$ and compute $k[x] \cap I$ and $k[y] \cap I$.
   (b) Find all solutions to the system.

(4) Consider the ideal,

$$I = \langle x^2 + y^2 + z^2 + 2, 3x^2 + 4y^2 + 4z^2 + 5 \rangle \subset k[x, y, z]$$

   (a) Find the elimination ideal $_1I$.
   (b) Compare $V(I)$ and $V(_1I)$ for $k = \mathbf{R}$ and explain.

You may use Macaulay 2 to solve problems (2)–(4); if you do, the relevant code is due together with your solutions.