Example 7.3.9

$$\ell_{2,j}(d) = \binom{2+d}{2} - \binom{2+d-j}{2}$$

$$= \frac{1}{2}(2+d)(1+d) - \frac{1}{2}(2+d-j)(2+d-j-1)$$

$$= \frac{1}{2}\left(d^2 + 3d + 2 - \left((d-j)^2 + 3(d-j) + 2\right)\right)$$

$$= \frac{1}{2}\left(d^2 + 3j - d^2 - j^2 + 2dj\right)$$

$$= \frac{1}{2}\left(2dj - j^2 + 3j\right)$$

$$= dj + \frac{3j - j^2}{2}$$

— x —

$d = 6$

$$6j + \frac{3j - j^2}{2} \le HF(R/i, 6) = 17$$

$j = 2$

With I saturated

$$\sum \Delta H(R/I, i) = \# \text{ pairs } I(x)$$

$$g \in I;$$

$$0 \to \frac{R}{(I : g)}(-j) \to R/I \to R/\langle I, g \rangle \to 0$$

$$\Delta H(R/I, i) = \Delta H(R/\langle I, g \rangle, i)$$
$$+ \Delta H(R/I : g, i-j)$$