$I \subseteq R = k[x_0, \ldots, x_n]$

$$HP(R/I, i) = \frac{a_r}{r!} i^r + \frac{a_{r-1}}{(r-1)!} i^{r-1} + \cdots$$

$r = \dim V(I)$

$a_r = \text{degree}\, V(I)$

codim $n-r$.

Last time: If f is R/I-regular, i.e. not a zero-divisor on R/I $(I : f) = I$, then

$$HP(R/\langle I, f\rangle) = \frac{a_r}{(r-1)!} i^{r-1} + \cdots$$

$\therefore$ $\dim V(I, f) = \dim V(I) - 1$

$\text{degree}\, V(I, f) = a_r = \deg V(I)$

Primary decomposition

$I = \bigcap Q_i$, $\sqrt{Q_i} = P_i$

$P_i = \sqrt{I : f_i}$

If $g \in P_i$, then $g^n f_i \in I$

$\therefore$ g is a zero-divisor on R/I.

What if $\sqrt{Q_j} = \langle x_0, \dots, x_n \rangle$

$I' = \bigcap_{i \neq j} Q_i \qquad V(I) = V(I')$

$HP(R/I', i) = HP(R/I, i)$

$HP(R/I, i) = HF(R/I, i) \quad i \gg 0$

$HP(R/I', i) = HF(R/I', i) \quad i \gg 0$

$I \subset I' \qquad I_i = I'_i \quad i \gg 0$

$$I' = \bigcap_{i \neq j} Q_i$$

$$\bigcup_{i \neq j} P_i \not\supseteq \langle x_0, \dots, x_n \rangle$$

by Prime Avoidance, so one can find a R/I'-reg. element.

<u>Def^n</u> Let M be a f.g. graded R-module. A seq. of homogeneous elt's $f_1, \dots, f_n$ in R is called an M-reg. (reg. for M) sequence if

f_1 is not a zero-divisor on M and f_i not a zero-divisor on $M/\langle f_1, \dots, f_{i-1} \rangle M$.

Such an ideal is called a complete intersection.

Bezout: f, g have no common components means that g is R/f regular

$(R = k[x_0, \ldots, x_2])$

If $\quad f = af'$

$\quad\quad g = ag'$

In $R/\langle f \rangle$ $\quad g f' = g' a f'$

$\quad\quad\quad\quad\quad\quad\quad\quad = g' f = 0$

<u>Ex</u> A computation like the proof of Bezout's theorem shows that the Hilbert poly of $R/\langle f_1, f_2, f_3 \rangle$ for a

regular sequence with

d_i = deg of f_i is

$$HP(R/\langle f_1, f_2, f_3\rangle) = d_1 d_2 d_3$$

$$0 \to R(-d_1 - d_2 - d_3) \xrightarrow{\begin{bmatrix} f_3 \\ -f_2 \\ f_1 \end{bmatrix}} \xrightarrow{\begin{bmatrix} f_2 & f_3 & 0 \\ -f_1 & 0 & f_3 \\ 0 & -f_1 & -f_2 \end{bmatrix}} \begin{matrix} R(-d_1) \\ \oplus \\ R(-d_2) \\ \oplus \\ R(-d_3) \end{matrix} \to R \to \frac{R}{\langle f_1 f_2 f_3\rangle} \to 0$$