MATH 5399-001 HOMEWORK DUE 28 MAR

Before 5 pm on 28 March turn in your handwritten or (much preferred) before Midnight on 28 March send me your TEX'ed solutions to the following problems.

(1) Exercise 4.2.6

(2) Exercise 4.4.6

(3) Consider ideal

$$I = \langle x^2 + y^2 + z^2 - 4, x^2 + 2y^2 - 5, xz - 1 \rangle \subset k[x, y, z]$$

(a) Find the elimination ideals ${}_1I$ and ${}_2I$.

(b) Find all solutions to the system

$$\begin{aligned} x^2 + y^2 + z^2 &= 4 \\ x^2 + 2y^2 &= 5 \\ xz &= 1 \end{aligned}$$

(4) Consider the system of equations

$$\begin{aligned} x^2 + 2y^2 &= 3 \\ x^2 + xy + y^2 &= 3 \end{aligned}$$

(a) Let I be the corresponding ideal in $k[x, y]$ and compute $k[x] \cap I$ and $k[y] \cap I$.

(b) Find all solutions to the system.

(5) Consider the ideal,

$$I = \langle x^2 + y^2 + z^2 + 2, 3x^2 + 4y^2 + 4z^2 + 5 \rangle \subset k[x, y, z]$$

(a) Find the elimination ideal ${}_1I$.

(b) Compare $V(I)$ and $V({}_1I)$ for $k = \mathbf{R}$ and explain.

You may use Macaulay 2 to solve problems (3)–(5); if you do, the relevant code is due by Midnight on 28 March.