MACAULAY 2 HOMEWORK DUE ON 14 FEB

Before Midnight on 14 February send me a Macaulay 2 file (extension m2) with the commands to solve the following problems. Please make `restart` the first command in your file.

(1) In the ring $\mathbb{Q}[x, y, z]/(xy - z^2)$ consider the ideal $P = \langle x, z \rangle$.
   (a) Decide if P is prime.
   (b) Decide if P^2 is primary.
   (c) Find a primary decomposition of P^2.

(2) In the polynomial algebra $\mathbb{Q}[x, y]$ consider the ideals

$$I = \langle x^2 - 1, y^2 - 1 \rangle \quad \text{and} \quad J = \langle x - y \rangle.$$

   (a) Decide if I is radical.
   (b) Compute the ideal quotient $I : J$.
   (c) Explain what goes on at the level of varieties.

(3) Find the Zariski closure of the following sets:
   (a) The projection of the hyperbola $V(\langle xy - 1 \rangle)$ in $\mathbb{R}^2$ onto the x-axis.
   (b) The boundary of the first quadrant in $\mathbb{R}^2$.
   (c) The set $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$.