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(1)

I. Wrap up groups.

II. Format of class

III. Ring Theory.

— x —

$U(8) = \{[1]_8, [3]_8, [5]_8, [7]_8\}$

$U(8) \neq \mathbb{Z}_2 \oplus \mathbb{Z}_2$ $\quad$ $U(8) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$

$U(8) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$

— x —

generic ring action

orbits

stabilizers

conjugation

conjugacy classes

centralizers

— x —

$H \leq G$

$K_G(H)$ conj. class

$G_H = \{g \in G \mid gHg^{-1} = H\}$

$= N_G(H)$ (2.3.20)

Normalizer

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$\mathbb{Z}_8 = (\mathbb{Z}_8, +)$ abelian group

$U(8) = (\{[1], [3], [5], [7]\}, \cdot)$

On $\mathbb{Z}$ we use $+$ and $\cdot$

<u>Defn</u> A set R with two operations $+$ and mult. is called a ring if

(1) $(R, +)$ is an abelian group.

(2) $\forall a, b \in R: ab \in R$ (closure)

(3) $\forall a, b, c \in R: (ab)c = a(bc)$ assoc.

(4) $\forall a, b, c \in R: a(b+c) = ab + ac$

$(b+c)a = ba + ca$

<u>Ex</u> $\mathbb{Z}, \mathbb{R}, \mathbb{Q}$ $M_{2\times 2}(\mathbb{R})$

<u>Ex</u> $R = \{\text{functions}: \mathbb{R} \to \mathbb{R}\}$

$(f+g)(x) := f(x) + g(x)$

$(fg)(x) := f(x)g(x)$

R is a ring.

<u>Ex</u> $\mathbb{Z}_n$ is a ring.

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(3)

<u>Prop.</u> Let R be a ring. The following hold:

(1) $a0 = 0 = 0a$ for all $a \in R$

(2) $(-a)b = -ab = a(-b)$, $\forall a, b \in R$

(3) $(-a)(-b) = ab$ $\forall a, b \in R$

(4) $(n \cdot a)(m \cdot b) = mn \cdot (ab)$ $\forall a, b \in R$ $\forall m, n \in \mathbb{Z}$

<u>Pf</u>: (1) $a0 + a0 = a(0+0)$
$$= a0$$
$$\therefore \quad a0 + a0 - a0 = a0 - a0$$
$$\therefore \quad a0 = 0$$

(2) $(-a)b + ab = (-a+a)b$
$$= 0b$$
$$= 0$$
$$\therefore (-a)b = -ab$$

(3) $(-a)(-b) = a(-(-b))$
$$= ab$$

4 By induction (p. 195)

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__Def^n__ A ring R is __commutative__ if $ab = ba$ for all $a, b \in R$.

__Def^n__ A ring R has a unit (unity) if there is an elt. $1 \in R$ s/t $1a = a = a1$ for all $a \in R$.

__Ex__ $\mathbb{Z}, \mathbb{Z}_n, \mathbb{R}, \mathbb{Q}, \mathbb{C}, -$ are all commutative with unit.

$M_{n \times n}(\mathbb{R})$ is a ring with unit

$1 = I_n$

__Def^n__ A non-empty subset of a ring R is a __subring__ if it is a ring under the operations from R

Thm (Subring Test). $S \neq \emptyset$ subset of a ring R is a subring if and only if

(1) $a - b \in S$ $\forall a, b \in S$

(2) $ab \in S$ $\forall a, b \in S$

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Ex $2\mathbb{Z} \subset \mathbb{Z}$ is a subring.

$a = 2x \quad b = 2y$

$a - b = 2x - 2y = 2(x-y) \in \overset{2\mathbb{Z}}{\cancel{2\in 2}}$

$ab = (2x)(2y) = 2(2xy) \in 2\mathbb{Z}.$

$2\mathbb{Z}$ is a ring w/o unit.

$z \in \mathbb{Z}, \ a \in 2\mathbb{Z}$

$za = z2x = 2(zx) \in 2\mathbb{Z}$

ideal

In $\mathbb{Z}_6$: $[2][3] = [0]$