

4/6/2020

①

- I. Wrap up groups.
- II. Format of class
- III. Ring Theory.

— X —

$$U(8) = \{[1]_8, [3]_8, [5]_8, [7]_8\}$$

$$U(8) \neq \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

$$U(8) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

$$U(8) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$$

— X —

generic ring action
orbits
stabilizers

conjugation
conjugacy classes
centralizers

— X —

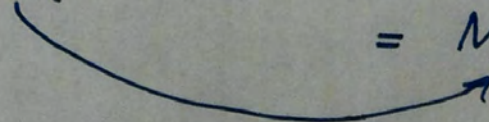
$$H \leq G$$

$K_G(H)$ conj.
class

$$G_H = \{g \in G \mid gHg^{-1} = H\}$$

$$= N_G(H) \quad (2.3.20)$$

Normalizer



②

4/6/2020

$$\mathbb{Z}_8 = (\mathbb{Z}_8, +) \text{ abelian group}$$

$$U(8) = (\{[1], [3], [5], [7]\}, \cdot)$$

On $\mathbb{Z}$ we use $+$ and $\cdot$.

Defⁿ A set R with two operations $+$ and mult. is called a ring if

- (1) $(R, +)$ is an abelian group.
- (2) $\forall a, b \in R: ab \in R$ (closure)
- (3) $\forall a, b, c \in R: (ab)c = a(bc)$ $\leftarrow$ assoc.
- (4) $\forall a, b, c \in R: a(b+c) = ab+ac$
 $(b+c)a = ba+ca$

Ex $\mathbb{Z}, \mathbb{R}, \mathbb{Q}, M_{2 \times 2}(\mathbb{R})$

Ex $R = \{\text{functions: } \mathbb{R} \rightarrow \mathbb{R}\}$

$$(f+g)(x) := f(x) + g(x)$$

$$(fg)(x) := f(x)g(x)$$

R is a ring.

Ex $\mathbb{Z}_n$ is a ring.

4/6/2020

(3)

Pwp. Let R be a ring. The following hold:

$$(1) \quad a0 = 0 = 0a \quad \text{for all } a \in R$$

$$(2) \quad (-a)b = -ab = a(-b), \quad \forall a, b \in R$$

$$(3) \quad (-a)(-b) = ab \quad \forall a, b \in R$$

$$(4) \quad (n \cdot a)(m \cdot b) = mn \cdot (ab) \quad \forall a, b \in R, \forall m, n \in \mathbb{Z}$$

Pf: (1) $a0 + a0 = a(0+0)$
 $= a0$

$$\therefore a0 + a0 - a0 = a0 - a0 \Leftrightarrow$$

$$\therefore a0 = 0$$

$$(2) \quad (-a)b + ab = (-a+a)b \\ = 0b \\ = 0$$

$$\therefore (-a)b = -ab$$

$$(3) \quad (-a)(-b) = a(-(-b)) \\ = ab$$

4/ By induction (p. 195)

(4)

4/6/2020

Defⁿ A ring R is commutative if $ab = ba$ for all $a, b \in R$.

Defⁿ A ring R has a unit (unity) if there is an elt. $1 \in R$ s.t. $1a = a = a1$ for all $a \in R$.

Ex $\mathbb{Z}, \mathbb{Z}_n, \mathbb{R}, \mathbb{Q}, \mathbb{C}, \dots$ are all commutative with unit.

$M_{n \times n}(\mathbb{R})$ is a ring with unit $1 = I_n$

Defⁿ A non-empty subset of a ring R is a subring if it is a ring under the operations from R

Thm (Subring Test). $S \neq \emptyset$ subset of a ring R is a subring if and only if

- (1) $a, b \in S \Rightarrow a+b \in S$
- (2) $a, b \in S \Rightarrow ab \in S$

(5)

4/6/2020

Ex $2\mathbb{Z} \subset \mathbb{Z}$ is a 'subring'.

$$a = 2x \quad b = 2y$$

$$a - b = 2x - 2y = 2(x - y) \in \frac{2\mathbb{Z}}{\cancel{2\mathbb{Z}}}$$

$$ab = (2x)(2y) = 2(2xy) \in 2\mathbb{Z}.$$

$2\mathbb{Z}$ is a ring w/o unit.

$$z \in \mathbb{Z}, a \in 2\mathbb{Z}$$

$$za = z2x = 2(zx) \in 2\mathbb{Z}$$

ideal

$$\text{In } \mathbb{Z}_6: [2][3] = [0]$$