MATH 3360 HOMEWORK ASSIGNMENT 17

DUE ON FRIDAY 1 MAY 2020

(1) Find polynomials $f(x)$ and $g(x)$ in $\mathbb{Z}_{12}[x]$ of degree 2 such that:
   (a) $f(x)g(x)$ had degree 3.
   (b) $f(x)g(x)$ had degree 2.
   (c) $f(x)g(x)$ had degree 1.
   (d) $f(x)g(x)$ had degree 0.

(2) Consider the polynomials $f(x) = 2x^4 + x^3 + 6x^2 + 1$ and $g(x) = x^2 + 3x - 1$ in $\mathbb{Z}[x]$. Find polynomials $q(x)$ and $r(x)$ with $r(x)$ of degree less than 2 such that
$$f(x) = q(x)g(x) + r(x)$$
holds.

(3) Let E and F be fields. Show that every ring homomorphism $\varphi \colon E \to F$ with $\varphi(1) \neq 0$ is 1-to-1.