

MATH 3360 HOMEWORK ASSIGNMENT 16

DUE ON TUESDAY 28 APRIL 2020

- (1) Let D be an integral domain with field of fractions F . Recall that addition and multiplication in F are given by

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \text{and} \quad \frac{a}{b} \frac{c}{d} = \frac{ac}{bd}.$$

Verify that the distributive laws hold in F .

- (2) Show that the map $\mathbb{Z}_{15} \rightarrow \mathbb{Z}_5$ given by $[x]_{15} \mapsto [x]_5$ is a ring homomorphism. Use this to show that the rings \mathbb{Z}_{15} and $\mathbb{Z}_3 \times \mathbb{Z}_5$ are isomorphic.

- (3) Let $\varphi: R \rightarrow S$ be a ring homomorphism. Show for every ideal J in S that $\varphi^{-1}(J)$ is an ideal in R that contains $\text{Ker } \varphi$.