MATH 3360 HOMEWORK ASSIGNMENT 15

DUE ON FRIDAY 24 APRIL 2020

(1) Let R be a ring with multiplicative identity 1. Show that the subset

$$R^c = \{r \in R \mid rx = xr \text{ for all } x \in R\}$$

is a subring of R.

(2) Let R be a ring. Recall that an element $x \in R$ is called an *idempotent* if $x^2 = x$ holds.

  (a) Show that in an integral domain, 0 and 1 are the only idempotents.
  (b) Show that if every element of R is idempotent, then R is commutative and $2x = 0$ holds for every $x \in R$.

(3) Let $\varphi\colon R \to S$ be a ring homomorphism. Set $S' = \varphi(R)$ and recall from 7.1.9 that is a subring of S.

  (a) Show for every ideal I in R that $\varphi(I)$ is an ideal in S'.
  (b) Show by example that $\varphi(I)$ may not be an ideal in S.