

MATH 3360 HOMEWORK ASSIGNMENT 13

DUE ON FRIDAY 17 APRIL 2020

- (1) For functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$ define the sum and product pointwise as follows:

$$(f + g)(x) = f(x) + g(x) \quad \text{and} \quad (fg)(x) = f(x)g(x).$$

Show that with these operations the set R of all functions $\mathbb{R} \rightarrow \mathbb{R}$ is a commutative ring with 1 (a unity in the terminology of the book). Decide if R is an integral domain.

- (2) Decide which of the following rings are fields; for those that are not, exhibit a non-zero element that does not have a multiplicative inverse.

- (a) \mathbb{Z}_2
- (b) \mathbb{Z}_{13}
- (c) $\mathbb{Z}_{13} \times \mathbb{Z}_4$
- (d) $\mathbb{Z}_{13} \times \mathbb{Z}_5$
- (e) \mathbb{Z}_{1747}
- (f) $\mathbb{Q} \times \mathbb{Q}$

- (3) Find all solutions to the equation $x^2 - 6x + 8 = 0$ in

- (a) \mathbb{Z}_{13}
- (b) \mathbb{Z}_{15}