MATH 3360 HOMEWORK ASSIGNMENT 6

DUE ON FRIDAY 6 MARCH 2020

(1) For each of the following product groups G determine the largest possible order of an element in G and give an example of an element $g \in G$ of maximal order.
   (a) $G = \mathbb{Z}_3 \times \mathbb{Z}_7$.
   (b) $G = \mathbb{Z}_6 \times \mathbb{Z}_{10}$.
   (c) $G = \mathbb{Z}_3 \times S_4$.

(2) Let G be a group and Z the center of G. Show that if the quotient group G/Z is cyclic, then G is abelian.

(3) Write $\mathbb{Z}_{30}$ as a direct sum of subgroups in two different ways.

(4) Find all possible group homomorphisms $\mathbb{Z}_6 \to \mathbb{Z}_8$.

(5) Let $\mathrm{M}_2(\mathbb{Z})$ be the set of 2×2 matrices with entries in $\mathbb{Z}$ and let I_2 denote the 2×2 identity matrix. Show that the matrices $A \in \mathrm{M}_2(\mathbb{Z})$ that satisfy $A^2 = I_2$ form a group that is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.