MATH 3360 HOMEWORK ASSIGNMENT 5

DUE ON FRIDAY 21 FEBRUARY 2020

(1) Prove that $\det\colon \mathrm{GL}_n(\mathbb{R}) \to \langle \mathbb{R} \setminus \{0\}, \cdot \rangle$ is a group homomorphism.

(2) Show that the collection of matrices of the form

$$\begin{pmatrix} x & -y \\ y & x \end{pmatrix} \quad \text{with} \quad (x, y) \in \mathbb{R} \times \mathbb{R} \setminus \{(0,0)\}$$

form a group under multiplication.

(3) The non-zero complex numbers, i.e. $\mathbb{C} \setminus \{0\}$, form a group under multiplication. Prove that it is isomorphic to the group from (2).

(4) (a) Let H and K be subgroups of G; show that $H \cap K$ is a subgroup of G. (b) Give an example of a group G with subgroups H and K such that $H \cup K$ is not a subgroup of G.

(5) Let G be a group and H a subgroup of G. Show that if $[G : H] = 2$, then $g^2 \in H$ for every $g \in G$.