MATH 3360 HOMEWORK ASSIGNMENT 4

DUE ON FRIDAY 14 FEBRUARY 2020

(1) Write the permutation

$$\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 2 & 6 & 3 & 8 & 7 & 1 \end{pmatrix}$$

(a) as a product of disjoint cycles and (b) as a product of transpositions. Decide if φ is odd or even.

(2) Determine the largest possible order of an element of S_{11} and give an example of an element of that order.

(3) Let $\varphi\colon G \to H$ be a group homomorphism and $f\colon H \to G$ a function. Show that if $f\varphi = \mathrm{Id}_G$ and $\varphi f = \mathrm{Id}_H$ hold, then f is a group homomorphism, and the groups G and H are isomorphic.

(4) Let G be a group that has proper subgroups of order 6, 8 and 12. What is the least possible order of G.

(5) List all elements of S_4 as products of disjoint cycles and determine the left and right cosets of A_4 in S_4.