MATH 3360 HOMEWORK ASSIGNMENT 3

DUE ON FRIDAY 7 FEBRUARY 2020

(1) Consider the permutations

$$\varphi = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 3\,5\,7\,1\,8\,6\,4\,2 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 7\,8\,5\,6\,1\,4\,3\,2 \end{pmatrix}$$

in S_8. Determine the permutation product $\tau\varphi$ and find the orbits of $\tau\varphi$.

(2) Let G be a group and $n \geqslant 2$ an integer. For elements $a_1, a_2, \ldots, a_n$ in G prove that one has

$$(a_1 a_2 \cdots a_n)^{-1} = a_n^{-1} \cdots a_2^{-1} a_1^{-1}.$$

Hint: Use induction on n.

(3) Let G be an abelian group. Show that if G has cyclic subgroups of order 6 and 15, then G has order at least 30.

(4) Prove that the collection H of 2×2 matrices of the form

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \quad \alpha \in \mathbb{R}$$

is a subgroup of $\mathrm{GL}_2(\mathbb{R})$.

(5) Let G be a cyclic group of order n and d a divisor in n. Show that the number of elements in G of order d is $\phi(n)$, where ϕ is Euler's function.