MATH 3360 HOMEWORK ASSIGNMENT 1

DUE ON FRIDAY 24 JANUARY 2020

(1) Find all integers that solve the congruence equation

$$x^2 + 3x + 2 \equiv_6 0\,.$$

(2) Let $n \in \mathbb{N}$ and let $\mathbb{Z}_n$ denote the equivalence classes of $\mathbb{Z}$ under the relation congruence mod n, i.e. $\equiv_n$. Prove that multiplication on $\mathbb{Z}_n$ given by

$$[x]_n \cdot [y]_n = [xy]_n$$

is a well-defined operation.

(3) Let $n \in \mathbb{N}$. Show that $\mathbb{Z}_n$ for $n \geqslant 2$ is not a group under multiplication as defined above. What happens for $n = 1$?

(4) Construct the group table for the group D_4 of symmetries of the square.

(5) Show that every set of cardinality 2 can be given an abelian group structure.