MATH 3310 HOMEWORK ASSIGNMENT 11

DUE ON FRIDAY 19 APRIL 2019

(1) Let A be a set. Prove that the relation $\approx$ on $\mathcal{P}(A)$ given by
$$X \approx Y \ \text{ if } \ |X| = |Y|$$
is an equivalence relation.

(2) For every natural number n set
$$a_n = \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} \ .$$
   (a) Compute the numbers a_1, a_2, a_3, and a_4.
   (b) Conjecture a closed form expression for a_n.
   (c) Prove the formula conjectured in part (b).

(3) Consider the relation F on $\mathbb{R} \times \mathbb{R}$ given by
$$F = \{(x, e^x) \mid x \in \mathbb{R}\} \ .$$
   (a) Decide if F is an equivalence relation.
   (b) Determine the inverse relation F^{-1}.

(4) Let R be the relation on $\mathbb{N}$ given by
$$a \ R \ b \ \text{ if } \ a|5b \text{ or } b|5a.$$
Decide if it is an equivalence relation.

(5) Let R be the relation on $\mathbb{Z}$ given by
$$a \ R \ b \ \text{ if } \ a^3 \equiv_4 b^3.$$
   (a) Prove that R is an equivalence relation.
   (b) Determine all the equivalence classes for R.