MATH 3310 HOMEWORK ASSIGNMENT 9

DUE ON FRIDAY 5 APRIL 2019

(1) Give a proof of the following statement:

The inequality $2^{n-1} \leqslant 2^n - 1$ *holds for all* $n \in \mathbb{N}$.

(2) Give a proof of the following statement:

The inequality $2n + 1 \leqslant 2^n$ *holds for all intgers* $n \geqslant 3$.

(3) Explain why the following argument is invalid.

Let x *and* y *be integers. If* $x = y$*, then one has* $xy = y^2$ *and hence* $x^2 - xy = x^2 - y^2$*. This may be rewritten as*

$$x(x - y) = (x + y)(x - y)\,,$$

and cancellation of common factors yields $x = x+y$*. Thus, for* $x = 1 = y$ *one gets* $1 = 2$.

(4) Prove or disprove each of the following statements.

(a) There exist integers x and y such that $12x + 9y = 100$ holds.
(b) Let $x, y \in \mathbb{R}$. If $x < y$ then $x^2 < y^2$.
(c) Let $n \in \mathbb{Z}$. If n is prime, then $2^n - 1$ is prime.
(d) Let $a, b \in \mathbb{Z}$. If a and b are even, then $4|(a^2 + b^2)$.

(5) Prove the following statement:

There are no integers x *and* y *with* $x^2 = 4y + 2$.