MATH 3310 HOMEWORK ASSIGNMENT 8

DUE ON FRIDAY 29 MARCH 2019

(1) Give a proof of the following statement:
Let A be a set. If A is well-ordered, then every subset of A is well-ordered.

(2) Give a proof of the following statement:
For sets A and B one has $A \cup B = A$ if and only if $B \subseteq A$.

(3) Consider the following induction argument.

> *In any subset of $\mathbb{Z}$ with only one element, all elements trivially have the same parity. Now, let $k \geqslant 1$ and assume that for every subset of $\mathbb{Z}$ with cardinality k all numbers in the subset have the same parity. Let B be a subset of $\mathbb{Z}$ with $|B| = k + 1$. Write $B = \{b_1, \ldots, b_k, b_{k+1}\}$. The subsets $B' = B - \{b_1\}$ and $B'' = B - \{b_{k+1}\}$ both have cardinality k, so in either set all elements have the same parity. Further, as B' and B'' have elements in common, the elements in B' and B'' have the same parity.*

The argument proves (mark all that apply):

(a) In a finite subset of $\mathbb{Z}$ all elements have the same parity.
(b) In a non-empty finite subset of $\mathbb{Z}$ all elements have the same parity.
(c) Nothing, the induction base is wrong.
(d) Nothing, the induction hypothesis is wrong.
(e) Nothing, the induction step is wrong.

(4) Prove that the following formula holds for all $n \in \mathbb{N}$:
$$\sum_{u=0}^{n} 2^u = 2^{n+1} - 1 \, .$$

(5) Prove the following statement:
For all real numbers x, y, and z one has $x^2 + y^2 + z^2 \geqslant xy + yz + zx$.