MATH 3310 HOMEWORK ASSIGNMENT 1

DUE ON FRIDAY 25 JANUARY 2019

(1) Describe each of the following sets by listing its elements within braces.
   (a) $A = \{n \in \mathbb{N} \mid -4 < n \leqslant 5\}$.
   (b) $B = \{n \in \mathbb{Z} \mid n^2 < 8\}$.
   (c) $C = \{x \in \mathbb{R} \mid x^2 + 9 = 0\}$.
   (d) $D = \{n \in \mathbb{Z} \mid -4 \leqslant n \leqslant 4\}$.
   (e) $E = \{x \in \mathbb{R} \mid x^2 - 5x + 6 = 0\}$.
   (f) $F = \{n \in \mathbb{Z} \mid 8 < n^3 < 100\}$.

(2) Consider the sets

$$A = \{x \in \mathbb{Q} \mid 2 < x \leqslant 4\},$$
$$B = \{n \in \mathbb{Z} \mid 2 \leqslant |n| < 4\},$$
$$C = \{x \in \mathbb{R} \mid x^2 - (2+\sqrt{2})x + 2\sqrt{2} = 0\}, \quad \text{and}$$
$$D = \{x \in \mathbb{Q} \mid x^2 - (2+\sqrt{2})x + 2\sqrt{2} = 0\}.$$

   (a) Describe the set B by listing its elements.
   (b) Determine the set $A \cap B$.
   (c) Give four examples of elements that belong to A and not to B.
   (d) Describe the set C by listing its elements.
   (e) Describe the set D in a different way than it is given.
   (f) Determine the cardinality of the sets B, C, and D.

(3) Determine the power set $\mathcal{P}(A)$ and its cardinality $|\mathcal{P}(A)|$ for
   (a) $A = \{a, b\}$.
   (b) $A = \{a, \{b\}, \varnothing\}$.
   (c) $A = \{a, \{b, \varnothing\}, \varnothing\}$.

(4) Decide whether the following statements are true of false.
   (a) If $\{a\} \in \mathcal{P}(A)$, then $\{a\} \notin A$.
   (b) If X and Y are finite sets with $|Y| = |X|+1$, then $|\mathcal{P}(Y)| \geqslant |\mathcal{P}(X)|+2$.
   (c) If W, X, Y, and Z are subsets of $\{x, y, z\}$ with $|W| = |X| = |Y| = |Z| = 2$, then at least two of the four sets are equal.

(5) Give examples of sets X, Y, and Z such that $X \neq Y$ but $X - Z = Y - Z$.