MATH 2360-012 WEEK 12

SECTIONS 7.1 AND 7.2; PAGES 347–367

Abstract. Let Q be a square matrix. If $\mathbf{v} \neq \mathbf{0}$ is a vector such that multiplying $\mathbf{v}$ by A yields a vector proportional to $\mathbf{v}$; i.e. $A\mathbf{v} = \lambda\mathbf{v}$, then one says that λ is an eigenvalue for A and $\mathbf{v}$ is a corresponding eigenvector. Thinking of multiplication by A as a linear transformation, it means that its effect on $\mathbf{v}$ is just scaling by the factor λ.

Section 7.1

Reading. Make sure that you understand the following:

(1) What eigenvectors and eigenvalues for matrices are.
(2) That to a given eigenvalue of a matrix there is an entire space of eigenvectors.
(3) That the eigenvalues for a matrix A are precisely the solutions to the characteristic equation $\det(\lambda I - A) = 0$, and that the eigenspace corresponding to an eigenvalue λ is the null space of the matrix $\lambda I - A$.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 3, 5, 11, 19, 23, and 41.

Section 7.2

Reading. Make sure that you understand the following:

(1) That an $n \times n$ matrix is *diagonalizable*, i.e. similar to a diagonal matrix, if and only if it has n linearly independent eigenvectors, and that is guaranteed to happen if it has n different eigenvalues.
(2) The procedure for diagonalizing a square matrix.
(3) That a strong motivation for diagonalizing matrices comes from the study of linear transformation. Given a linear transformation $T \colon V \to V$ and a choice of basis for V, the transformation is represented by a square matrix. Diagonalizing that matrix means finding a different basis for V such that T with respect to that basis is represented by a simple square matrix: a diagonal one.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1, 5, 7, 19, and 25.

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