

MATH 2360-012 WEEK 10

SECTIONS 6.1 AND 6.2; PAGES 297–319

ABSTRACT. A *linear transformation* is a simple, but extremely useful, type of function. It maps from one vector space to another and respects the addition and multiplication by scalars. Every matrix defines a linear transformation.

SECTION 6.1

Reading. Make sure that you understand the following:

- (1) The definition of a *linear transformation*.
- (2) That an $m \times n$ matrix A defines a linear transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ given by $T(\mathbf{x}) = A\mathbf{x}$ for vectors $\mathbf{x} \in \mathbb{R}^n$.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1–4, 11, 17, 23, and 35.

SECTION 6.2

Reading. Make sure that you understand the following:

- (1) Given a linear transformation $T: V \rightarrow W$, the *kernel* of T is a subspace of V and the *range*—also called the *image*—of T is a subspace of W .
The kernel consists of the vectors in V that map to $\mathbf{0}$, and the range consists of the vectors in W that get “hit” by T .
- (2) For a linear transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ given by $T(\mathbf{x}) = A\mathbf{x}$, the kernel is the null space of A and the range is the column space of A . This explains why the dimension of the kernel and range of a linear transformation is called its *nullity* and *rank*, respectively.
- (3) An *isomorphism* $V \rightarrow W$ of vector spaces is a linear transformation that is one-to-one (the kernel is $\{\mathbf{0}\}$) and *onto* (the range is W).
- (4) Isomorphic vector spaces have the same dimension.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 3–7, 13, 53.