MATH 2360-012 WEEK 7

SECTIONS 4.3 AND 4.4; PAGES 168–185

Abstract. A subset U of a vector space V is called a *subspace* if it is a vector space in its own right. The xy-plane, for example, is a subspace of $\mathbb{R}^3$. We are aiming for simple descriptions of vector spaces. The goal is to identify within a vector space V a list of vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$ such that every vector in V can be built from $\mathbf{u}_1, \dots, \mathbf{u}_n$ in a unique way.

Section 4.3

Reading. Make sure that you understand the following:

(1) A subspace of a vector space is a non-empty subset that is "closed" under addition and scalar multiplication.
(2) In every vector space the subset consisting is the zero-vector is a subspace, and every vector space is a subspace of itself.
(3) The subspaces of $\mathbb{R}^2$ are: $\{\mathbf{0}\}$, $\mathbb{R}^2$, and all lines through the origin.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1, 3, 7, 15, 29, and 39.

Section 4.4

Reading. Make sure that you understand the following:

(1) A vector $\mathbf{v}$ in a vector space V is a *linear combination* of a family $\mathbf{u}_1, \dots, \mathbf{u}_n$ of vectors in V if there exist constants $c_1, \dots, c_n$ such that
$$\mathbf{v} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n\,.$$
(2) A *spanning set* for a vector space V is a set of vectors in V that is "big enough" to allow every other vector in V to be written as a linear combination of those in the set.
(3) In general, the *Span* of a set of vectors in a space V is the collection of all vectors that can be written as linear combinations of those in the set; that collection is a subspace of V.
(4) A set of vectors $\mathbf{u}_1, \dots, \mathbf{u}_n$ is *linearly independent* if choosing $c_1, \dots, c_n$ all equal to 0 is the only way to write the zero-vector as a linear combination,
$$\mathbf{0} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n\,.$$
(5) Two vectors $\mathbf{u}$ and $\mathbf{v}$ are linearly independent if and only if there is no constant c with $\mathbf{u} = c\mathbf{v}$ or $\mathbf{v} = c\mathbf{u}$.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 3, 5, 9, 25, 33, and 45.

Date: September 30, 2020.