MATH 2360-012 WEEK 6

SECTIONS 4.1 AND 4.2; PAGES 151–167

ABSTRACT. We now move towards the abstract notion of a vector space: It is "something" that behaves a lot like $\mathbb{R}^3$ though, at first glance, it may look nothing like euclidean space. Many basic questions about vector spaces are conveniently phrased in the language of matrices and matrix equations. Moreover, the language of vector spaces will give us a way to geometrically interpret the solutions to systems of linear equations, also when dealing with more than three variables.

SECTION 4.1

Reading. Make sure that you understand the following:

(1) Every point P in $\mathbb{R}^2$ (the euclidean plane) determines a vector $\mathbf{u}$, namely the one that terminates at P when placed so that its root is at the origin. In practice, we often identify a point and the vector it defines; for computational purposes $\mathbf{u}$ is represented by a pair of numbers (u_1, u_2), namely the coordinates of P.

In the same way, every point in $\mathbb{R}^3$ determines a vector, which for computational purposes is represented by a triple of numbers (u_1, u_2, u_3).

(2) One can do the exact same computations with n-tuples $(u_1, u_2, \ldots, u_n)$, they are called vectors in $\mathbb{R}^n$ (n-space).

(3) A vector $\mathbf{x}$ in $\mathbb{R}^n$ is a *linear combination* of a family $\mathbf{u}_1, \ldots, \mathbf{u}_n$ of vectors in $\mathbb{R}^n$ if there exist constants $c_1, \ldots, c_n$ such that $\mathbf{x} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n$.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 22, 29, and 45.

SECTION 4.2

Warning. Up to this point, the course has focused on solving systems of linear equations. We are now changing gears and will spend most of our time on vector spaces. Most of the problems will still be solved by constructing a matrix, putting it on Row Echelon Form, and then interpreting what it tells us. Thus, the computational techniques you have acquired will serve you for the rest of the course.

Reading. Make sure that you understand the following:

(1) A vector space is a set V with two operations: addition and scalar multiplication. These two operations have to behave (and interact) like addition and scalar multiplication on $\mathbb{R}^n$.

(2) It is not always easy to visualize vectors as arrows in some space; for example, the set of all continuous functions defined on the real number line form a vector space. What "arrow" exactly is $\cos x$?

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 17–18, 23–26, and 30.

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