

MATH 2360-012 WEEK 5

SECTIONS 3.3 AND 3.4; PAGES 126–143

ABSTRACT. The determinant of a square matrix betrays if it is invertible. It is possible—but in practice often cumbersome—to find the inverse of a matrix by computation of determinants alone; that is, without doing any row operations.

SECTION 3.3

Reading. Make sure that you understand the following:

- (1) The determinant of a matrix product is the product of the determinants,
$$\det AB = (\det A)(\det B) .$$
- (2) A square matrix is invertible if and only if it has non-zero determinant.
- (3) A matrix is invertible if and only if it is a product of elementary matrices.
- (4) A matrix is invertible if and only if its reduced row echelon form is the identity matrix.
- (5) A system of n linear equations in n variables can be written as a matrix equation $AX = B$, where A is an $n \times n$ matrix and B and X are $n \times 1$ columns. The system has a unique solution if and only if A is invertible, and then the solution is $X = A^{-1}B$.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 2–4, 8, and 13.

SECTION 3.4

Reading. Make sure that you understand the following:

- (1) Given a square matrix A , the matrix of cofactors of A has as its entry in row i and column j the number $C_{ij} = (-1)^{i+j}M_{ij}$, where the minor M_{ij} is the determinant of the matrix obtained from A by deleting row i and column j .
- (2) The *adjoint* matrix of A , written $\text{adj } A$ is the transpose of the matrix of cofactors.
- (3) If A is a square matrix with $\det A \neq 0$, then

$$A^{-1} = \frac{1}{(\det A)} \text{adj } A .$$

- (4) For 2×2 matrices the formula above is worth memorizing,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ with } d = a_{11}a_{22} - a_{12}a_{21} \neq 0 \text{ has } A^{-1} = \frac{1}{d} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} .$$

- (5) The determinant can be interpreted geometrically.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1–4, 18, and 31.