MATH 2360-012 WEEK 4

SECTIONS 3.1 AND 3.2; PAGES 109–125

ABSTRACT. A system of n linear equations in n variables has a unique solution if and only if the corresponding matrix of coefficients is nonsingular, also called *invertible*. Thus, we want to know how to decide if an $n \times n$ matrix is invertible.

We already know how to use Gauss–Jordan elimination for this purpose. Computing the *determinant* is another way. (Spoiler alert! An $n \times n$ matrix is invertible if and only if its determinant is not 0). At this moment, the determinant is just a number that one associates to a *square*, i.e. $n \times n$ matrix. It plays a central role later in this course, so give it a chance.

SECTION 3.1

Reading. Make sure that you understand the following:

(1) A 1×1 matrix A has one entry a, i.e. $A = [a]$, and the determinant of A is a.
(2) The determinant of an $n \times n$ matrix is by cofactor expansion computable in terms of determinants of $(n-1) \times (n-1)$ matrices. This means that we can "bootstrap" our way up from 1×1 matrices.
(3) It is, nevertheless, worth the effort to memorize how to compute the determinant of a 2×2 matrix; larger ones not so much.
(4) The determinant of a triangular matrix is the product of the diagonal entries.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1–5, 15, 21, and 28.

SECTION 3.2

Reading. Make sure that you understand the following:

(1) The most common row operation, adding a multiple of one row to another, does not affect the determinant.
(2) Interchanging two rows changes the sign of the determinant.
(3) Multiplying a row by a constant c has the effect of multiplying the determinant by c.
(4) The effect of scalar multiplication on the determinant depends on the size of the matrix. If A is an $n \times n$ matrix, then one has $\det(cA) = c^n \det(A)$.
(5) A matrix with a zero row/column has determinant 0.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 4–9, 29, and 34.

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