MATH 2360-012 WEEK 3

SECTIONS 2.3 AND 2.4; PAGES 62–83

Abstract. A system of linear equations can be written as a single matrix equation $AX = B$; the goal is now to solve the system by solving the matrix equation the same way one would solve an equation $ax = b$ of numbers, namely by dividing by a. It is not always possible, and that is because a system of equations may have no solution, a unique solution, or an infinite family of solutions. Only in the second case can one expect to find the solution by "division"; and it is possible.

Division is multiplication by the reciprocal/inverse. The key to determining whether a matrix has an *inverse* and, if so, finding it, is to recast row operations as matrix multiplication. This leads to *elementary matrices*. Thus, the problem of solving a system of equations has been turned into a problem in matrix algebra.

Section 2.3

Reading. Make sure that you understand the following:

(1) Not every matrix has an inverse, not even every non-zero matrix.
(2) Only a square matrix can have an inverse, and it is unique if it exists.
(3) Gauss–Jordan Elimination can be used to decide if a matrix has an inverse.
(4) The cancellation properties of inverse matrices.
(5) A system of n linear equations in n variables has a unique solution if and only if the coefficient matrix has an inverse.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 2, 6, 10, 14, and 25.

Section 2.4

Reading. Make sure that you understand the following:

(1) Every elementary matrix E is obtained from the identity matrix by performing a single row operation, and multiplication on the left by E on another matrix A is tantamount to performing the same row operation on A.
(2) A matrix is invertible if and only if it is a product of elementary matrices.
(3) The LU -factorization. It makes things easier down the road because (upper or lower) triangular matrices are easy to work with.

Suggested problems. To verify that you have understood the material, solve the following problems at the end of the section: 1–8, 10, 11, 24, 25, and 43.

Applications

Study Example 1 in Section 2.5 and try your hand at exercises 1, 2 and 5 in that section.

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