

1. (25%) Consider the curve $\mathcal{C}$ given by the vector valued function

$$\mathbf{R}(t) = \langle t, t^2 \rangle = t\mathbf{i} + t^2\mathbf{j}, \quad t \geq 0.$$

Find the unit tangent vector $\mathbf{T}(t)$ for $\mathcal{C}$.

2. (25%) Rewrite the integral

$$\int_0^1 \int_0^{2x} f(x, y) \, dy \, dx$$

with the order of integration reversed.

3. (25%) Rewrite the function

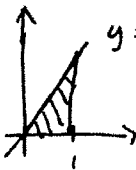
$$f(x, y, z) = x + ze^{x^2+y^2}$$

in cylindrical coordinates.

4. (25%) Compute curl of the vector field

$$\mathbf{F}(x, y, z) = \langle y^2 + z, x^2 + z, x^2 + y \rangle = (y^2 + z)\mathbf{i} + (x^2 + z)\mathbf{j} + (x^2 + y)\mathbf{k}.$$

① $\bar{\mathbf{R}}'(t) = \langle 1, 2t \rangle, \quad \|\bar{\mathbf{R}}'(t)\| = \sqrt{1+4t^2}, \quad \bar{\mathbf{T}}(t) = \frac{1}{\sqrt{1+4t^2}} \langle 1, 2t \rangle$

②  $y=2x \Leftrightarrow x=\frac{1}{2}y$ $\int_0^1 \int_0^{2x} f(x, y) \, dy \, dx = \int_0^2 \int_{\frac{1}{2}y}^1 f(x, y) \, dx \, dy$

③ $f(r, \theta, z) = r \cos \theta + ze^{r^2}$

④ $\text{curl } \bar{\mathbf{F}} = \nabla \times \bar{\mathbf{F}} = \begin{vmatrix} \bar{\mathbf{i}} & \bar{\mathbf{j}} & \bar{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2+z & x^2+z & x^2+y \end{vmatrix}$

$$= (1-1)\bar{\mathbf{i}} - (2x-1)\bar{\mathbf{j}} + (2x-2y)\bar{\mathbf{k}} = \langle 0, 1-2x, 2x-2y \rangle$$