Quiz 4
Math 2350
Fall 2007

Name and section Solution

Consider the function

$$f(x,y) = x^3 + xy + y^3$$

defined everywhere in the plane $\mathbf{R}^2$.

1. (25%) Find the gradient $\nabla f(x,y)$.
2. (30%) Find the critical points of f (there are two).
3. (20%) Find the second order derivatives f_{xx}, f_{yy}, and f_{xy}.
4. (25%) Classify each of the critical points as a relative minimum, a relative maximum, or a saddle point.

① $\nabla f(x,y) = \langle 3x^2+y,\ x+3y^2 \rangle$

② ∇f is defined everywhere, so the critical points are the ones where $\nabla f = \bar{0}$.

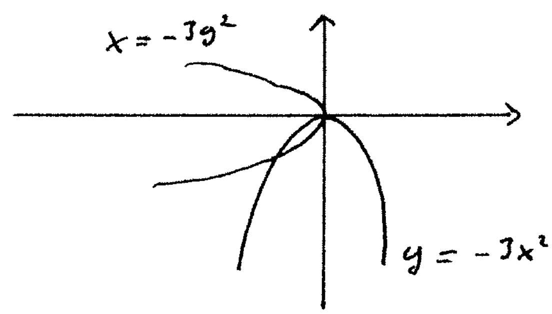
$3x^2 + y = 0 \iff y = -3x^2$

$0 = x + 3y^2 = x + 3(-3x^2)^2 = x + 27x^4 = x(1 + 27x^3)$

has two real solutions

$x = 0$ and $x = -\frac{1}{3}$

so the critical points are $(0,0)$ and $(-\frac{1}{3}, -\frac{1}{3})$



③ $f_{xx}(x,y) = 6x$

$f_{yy}(x,y) = 6y$

$f_{xy}(x,y) = 1$

④ At $(0,0)$ the discriminant is $(0)(0) - 1^2 = -1 < 0$ so it is a saddle point

At $(-\frac{1}{3}, -\frac{1}{3})$ the discriminant is $(-2)(-2) - 1^2 = 3 > 0$ and $f_{xx} = -2 < 0$ so it is a relative maximum