Quiz 2
Math 2350
Fall 2007

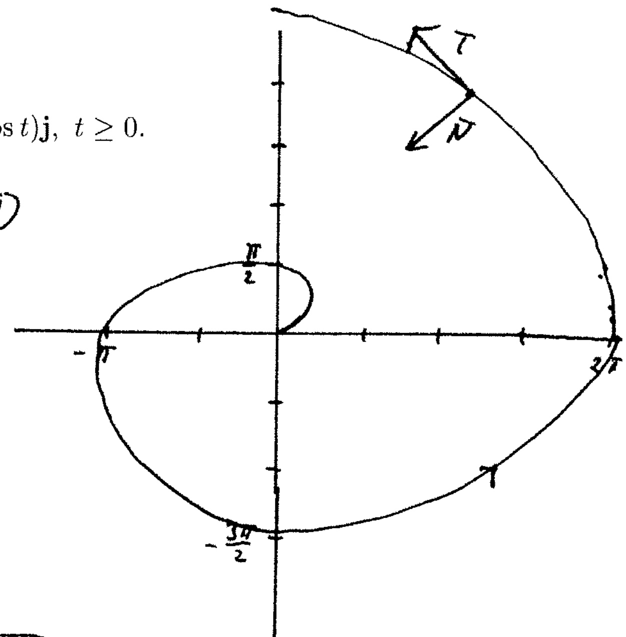
Name and section Solution

Consider the curve $\mathcal{C}$ given by the vector valued function

$$\mathbf{R}(t) = \langle t\cos t, t\sin t\rangle = (t\cos t)\mathbf{i} + (t\cos t)\mathbf{j},\ t \geq 0.$$

1. (20%) Sketch $\mathcal{C}$.
2. (25%) Find the first derivative $\mathbf{R}'(t)$.
3. (25%) Find unit tangent vector $\mathbf{T}(t)$ for $\mathcal{C}$.
4. (20%) Find the principal normal vector $\mathbf{N}(t)$ for $\mathcal{C}$.
5. (10%) Is $\mathcal{C}$ a smooth curve? Why/why not?

①



② $\bar{R}(t) = \langle \cos t - t\sin t,\ \sin t + t\cos t \rangle$

③ $\|\bar{R}(t)\| = \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}$

$= \sqrt{\cos^2 t + t^2\sin^2 t - 2t(\cos t)(\sin t) + \sin^2 t + t^2\cos^2 t + 2t(\cos t)(\sin t)}$

$= \sqrt{\cos^2 t + \sin^2 t + t^2\sin^2 t + t^2\cos^2 t} = \sqrt{1+t^2}$

$\bar{T}(t) = \dfrac{\bar{R}'(t)}{\|\bar{R}'(t)\|} = \dfrac{1}{\sqrt{1+t^2}}\langle \cos t - t\sin t,\ \sin t + t\cos t\rangle$

④ Since C is a curve in the plane, the unit vector $\bar{N}$ that is orthogonal to $\bar{T}$ is unique up to sign (± 1). From ① it is clear that $\bar{N}$ is obtained by rotating $\bar{T}$ $\frac{\pi}{2}$ counterclockwise, so

$$\bar{N} = \frac{1}{\sqrt{1+t^2}}\langle -(\sin t + t\cos t),\ \cos t - t\sin t\rangle$$

⑤ C is smooth as $\|\bar{R}(t)\| \geq 1$ so $\bar{R}(t)$ is never $\langle 0,0\rangle$, the 0-vector.