

This set of problems is similar in difficulty and length to the final exam. Try to solve the problems in two hours without consulting the text book, except for the tables of integrals and Laplace transforms.

Before 10:30 am on Tuesday (April, 26), you may hand in your written solution to any five of these problems. This will count as an extra set of homework and allow you to drop a third set.

1. Consider the differential equation

$$(*) \quad \frac{dy}{dx} = \sqrt{x - y}.$$

- (a) Verify that the function

$$y(x) = x - 1$$

is a solution to $(*)$ on the whole real line.

- (b) There is an existence and uniqueness theorem for solutions to first order differential equations. Does this theorem guarantee existence of a solution to $(*)$ with $y(2) = 2$? If so, is it unique?

2. Consider the differential equation

$$(*) \quad yy' = \frac{x}{2\sqrt{x^2 - 9}}.$$

- (a) Find, in implicit form, the general solution to $(*)$.

- (b) Find, in **explicit** form, a particular solution to $(*)$ with $y(5) = 8$.

3. Solve the initial value problem

$$\frac{dy}{dx} = (1 - y) \cos x; \quad y(\pi) = 2.$$

4. Consider the autonomous differential equation

$$(*) \quad \frac{dx}{dt} = 7x - 10 - x^2.$$

- (a) Find the critical points.

- (b) For each of the critical points, determine whether it is stable or unstable.

- (c) Draw the phase diagram for $(*)$.

5. Find the general solution to the nonhomogeneous differential equation

$$y'' + 2y' - 3y = 1 + xe^x.$$

6. Consider the following system of differential equations

$$(**) \quad \frac{dx_1}{dt} = x_1 - 2x_2 - 5t \quad \text{and} \quad \frac{dx_2}{dt} = 2x_1 + x_2 - 9$$

(a) Write the system (**) in matrix notation.

(b) Find the solution to (**) with $x_1(0) = 3$ and $x_2(0) = 5$.

7. Find the general solution to the system

$$\mathbf{x}' = \begin{bmatrix} 25 & 12 & 0 \\ -18 & -5 & 0 \\ 6 & 6 & 13 \end{bmatrix} \mathbf{x}.$$

8. Use Laplace transforms to solve the initial value problem

$$x'' + 8x' + 15x = 0; \quad x(0) = 2 \text{ and } x'(0) = -3.$$