Sample questions for exam 2 math 1452 KEY

1. Find he following indefnite integral $\int (x^2+x)e^{2x}\,dx = \frac{1}{2}x^2e^{2x}$
   (a) $\frac{3}{2}x^2e^{2x} - xe^{2x} + \frac{1}{2}e^{2x} + C$
   (b) (correct) $\frac{1}{2}x^2e^{2x} + C$
   (c) $\frac{3}{2}x^2e^{2x} + xe^{2x} + \frac{1}{2}e^{2x} + C$
   (d) $\left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\left(\frac{1}{2}e^{2x}\right) + C$
   (e) none of the above

2. $\int \frac{1}{(x+2)(x-1)}dx$
   (a) (correct)$-\frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$
   (b) $\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x-1| + C$
   (c) $\frac{2}{3}\ln|x-1| - \frac{1}{3}\ln|x-1| + C$
   (d) $\ln|x-1|\ln|x-1| + C$
   (e) None

3. What is the right partial fractions form for the function $\frac{x}{(x^2+2x+5)^2(x-1)^2}$?
   (a) $\frac{A}{(x^2+2x+5)^2} + \frac{B}{(x-1)^2}$
   (b) $\frac{Ax+B}{(x^2+2x+5)^2} + \frac{C}{(x-1)^2}$
   (c) $\frac{Ax+B}{x^2+2x+5} + \frac{Cx+D}{(x^2+2x+5)^2} + \frac{E}{(x-1)^2}$
   (d) $\frac{A}{x^2+2x+5} + \frac{B}{(x^2+2x+5)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$
   (e) (correct) none of the above

4. If $\frac{10}{(x^2+2x+2)(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+2x+2}$ determine the correct value of B.
   (a) 1 (correct)
   (b) 3
   (c) -3
   (d) 3/5
   (e) none of the above

5. If $I = \int e^x\cos(2x)\,dx$ then which of the following is true and shows up near the end of the calculaion when one is determining the integral?
   (a) $I = e^x\cos 2x + 2e^x\sin 2x$
   (b) $I = 2e^x\cos 2x + e^x\sin 2x - 4I$

(c) $I = 2e^x \cos 2x + e^x \sin 2x - 3I$

(d) $I = e^x \cos 2x + 2e^x \sin 2x - 4I$ (correct)

(e) None

6. Which of the following is true assuming all integral and derivatives exist?

(a) $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$

(b) $\int_a^b f(x)g(x)dx = \int_a^c f(x)dx \int_c^b g(x)dx$

(c) (correct) $\int_a^b f(x)g'(x)dx = [f(b)g(b) - f(a)g(a)] - \int_a^b f'(x)g(x)dx$ (this is just a version of integration by parts)

(d) $\int_a^b f(x)g'(x)dx = [f(b)g(b) + f(a)g(a)] - \int_a^b f(x)g'(x)dx$

(e) $\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right) g(x) + f(x) \left(\int_a^b g(x)dx\right)$

7. Which comparison is valid and shows that $\int_1^\infty \frac{2}{x^2+\sqrt{x}} dx$ converges?

(a) $\int_1^\infty \frac{1}{x^2} dx \leq \int_1^\infty \frac{2}{x^2+\sqrt{x}} dx$

(b) (correct) $\int_1^\infty \frac{2}{x^2+\sqrt{x}} dx \leq 2\int_1^\infty \frac{1}{x^2} dx$

(c) $\int_1^\infty \frac{2}{x^2+\sqrt{x}} dx \leq 2\int_1^\infty \frac{1}{\sqrt{x}} dx$

(d) $\int_1^\infty \frac{1}{2\sqrt{x}} dx \leq \int_1^\infty \frac{2}{x^2+\sqrt{x}} dx$

(e) None

8. Which comparison is valid and shows that $\int_0^1 \frac{1}{x^2+\sqrt{x}} dx$ converges?

(a) $\int_0^1 \frac{1}{x^2} dx \leq \int_0^1 \frac{1}{x^2+\sqrt{x}} dx$

(b) $\int_0^1 \frac{1}{2\sqrt{x}} dx \leq \int_0^1 \frac{1}{x^2+\sqrt{x}} dx$

(c) $\int_0^1 \frac{1}{x^2+\sqrt{x}} dx \leq \int_0^1 \frac{1}{2\sqrt{x}} dx$

(d) (correct) $\int_0^1 \frac{1}{x^2+\sqrt{x}} dx \leq \int_0^1 \frac{1}{\sqrt{x}} dx$

(e) none

9. Which comparison is valid and shows that $\int_0^1 \frac{1}{1+\sqrt{x}} dx$ diverges

(a) $\int_0^1 \frac{1}{1+\sqrt{x}} dx \leq \int_0^1 \frac{1}{\sqrt{x}} dx$

(b) $\int_0^1 \frac{1}{\sqrt{x}} dx \leq \int_0^1 \frac{1}{1+\sqrt{x}} dx$

(c) $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}} dx \leq \int_0^1 \frac{1}{1+\sqrt{x}} dx$

(d) $\int_0^1 \frac{1}{1+\sqrt{x}} dx \leq \int_0^1 \frac{1}{x} dx$

(e) (correct) none (since in fact $\int_0^1 \frac{1}{1+\sqrt{x}} dx$ converges by comparison $\int_0^1 \frac{1}{1+\sqrt{x}} dx \leq \int_0^1 \frac{1}{\sqrt{x}} dx$

10. Evaluate the integral $\int_0^1 x^2 e^{2x} dx$

(a) (correct) $\frac{1}{4}e^2 - \frac{1}{4}$
(b) $\frac{1}{4}e^2 + \frac{1}{4}$
(c) $\frac{1}{2}e^2 + \frac{1}{4}$
(d) $\frac{1}{2}e^2 - \frac{1}{4}$
(e) none

11. Evaluate $\int_1^\infty xe^{-x} dx$

(a) (correct) $2e^{-1}$
(b) 2
(c) It diverges
(d) e^{-1}
(e) None of the above is correct

12. Which of the following integrals is *not* improper?

(a) $\int_1^\infty \frac{1}{(x-1)(x-3)^2} dx$
(b) $\int_1^2 \frac{1}{(x-1)(x-3)^2} dx$
(c) $\int_2^3 \frac{1}{(x-1)(x-3)^2} dx$
(d) (correct) $\int_4^5 \frac{1}{(x-1)(x-3)^2} dx$
(e) $None$

13. Find the antiderivative $\int \sqrt{4-x^2} dx$
Use the sub $x = 2\sin\theta$

$$
\begin{aligned}
\int \sqrt{4-x^2} dx &= \int (2\cos\theta)\, 2\cos\theta d\theta \\
&= 4 \int \cos^2\theta d\theta \\
& \quad 4 \int (\frac{1}{2} + \frac{1}{2}\cos 2\theta) d\theta \\
& \quad 2 \int (1 + \cos 2\theta) d\theta \\
& \quad 2\theta + \sin 2\theta + C \\
&= 2\theta + 2\cos\theta \sin\theta + C \\
&= 2\sin^{-1}(x/2) + \frac{x\sqrt{4-x^2}}{2} + C
\end{aligned}
$$

check $\frac{d}{dx}\left(2\sin^{-1}(x/2)+\frac{x\sqrt{4-x^2}}{2}\right)=\sqrt{4-x^2}$

(a) $\sqrt{4-x^2}-\tan^{-1}\frac{2}{\sqrt{4-x^2}}+C$
(b) $\sqrt{4-x^2}+C$
(c) $\sqrt{4-x^2}-\tan^{-1}\frac{1}{\sqrt{4-x^2}}+C$
(d) $\frac{1}{2}\sqrt{4-x^2}-\tan^{-1}\frac{1}{\sqrt{4-x^2}}+C$
(e) None (correct)

14. What is the correct/best trigonometric substitution for $\int \frac{x^2}{(9-x^2)^{3/2}}dx$?

(a) (correct) $x=3\sin\theta$
(b) $x=\frac{1}{3}\sin\theta$
(c) $x=\frac{1}{9}\sin\theta$
(d) $x=\frac{1}{3}\sin\theta$
(e) none

15. What is the correct/best trigonometric substitution for $\int \frac{1}{(4x^2-9)^{3/2}}dx$?

(a) $x=3\sin\theta$
(b) $x=2\sec\theta$
(c) $x=(3/2)\sin\theta$
(d) (correct) $x=(3/2)\sec\theta$
(e) none of the above

16. What is the correct/best trigonometric substitution for $\int \frac{1}{(x^2-4x)^{3/2}}dx$ for $x>4$?

$$\begin{aligned}\int \frac{1}{(x^2-4x)^{3/2}}dx &= \int \frac{1}{((x-2)^2-4)^{3/2}}dx = \int \frac{1}{\left(\sqrt{(x-2)^2-4}\right)^3}dx \\ x-2 &= 2\sec\theta \\ x &= 2\sec\theta+2\end{aligned}$$

(a) $x=2\cos\theta$
(b) $x-1=2\cos\theta$
(c) $x=2\tan\theta$
(d) $x-1=2\tan\theta$
(e) none—>$x=2\sec\theta+2$ (this was missing the correct choice "none")

17. For what values of p does the following improper integral converge? $\int_1^2 \frac{1}{(x-1)^p}$

(a) $p > 2$
(b) $p \geq 1$
(c) $p \leq 1$
(d) (correct) $p < 1$
(e) none are correct

18. (this one was corrected 1/2–>1/3 to avoid negatives under a square root) Which of the following give the correct interpretation of the improper integral as a limit?

    (a) $\int_0^3 \frac{1}{(x-1)^{1/3}} dx = \lim_{a \to 0^+} \int_a^3 \frac{1}{(x-1)^{1/3}} dx$
    (b) $\int_0^3 \frac{1}{(x-1)^{1/3}} dx = \lim_{a \to 1^-} \int_a^3 \frac{1}{(x-1)^{1/3}} dx$
    (c) (correct) $\int_0^3 \frac{1}{(x-1)^{1/3}} dx = \lim_{a \to 1^-} \int_0^a \frac{1}{(x-1)^{1/3}} dx + \lim_{a \to 1^+} \int_a^3 \frac{1}{(x-1)^{1/3}} dx$
    (d) $\int_0^3 \frac{1}{(x-1)^{1/3}} dx == \lim_{a \to 0^-} \int_0^a \frac{1}{(x-1)^{1/3}} dx + \lim_{a \to 0^+} \int_a^3 \frac{1}{(x-1)^{1/3}} dx$
    (e) None are correct

19. After making the correct trig substitution, the integral $\int \frac{1}{x^4\sqrt{9-x^2}}$ becomes and integral in a new vaiable θ. The integral is which of the following?

    (a) (correct) $\frac{1}{81} \int \csc^4(\theta) d\theta$
    (b) $\frac{1}{81} \int \sin^4(\theta) d\theta$
    (c) $\frac{1}{9} \int \csc^4(\theta) d\theta$
    (d) $\frac{1}{9} \int \sec^3(\theta) d\theta$
    (e) none of the above

20. What u-substitution is appropriate for the integral $\int \sec^3(x) \tan^3(x)\, dx$?

    (a) $u = \tan(x)$
    (b) $u = \tan(x)$
    (c) $u = \sec^2(x)$
    (d) (correct) $u = \sec(x)$
    (e) none of the above

21. Find the integral $\int_0^\pi \sin^3(x) dx$

    (a) 4
    (b) $\frac{4}{3}$
    (c) 3
    (d) $\frac{3}{4}$
    (e) None of the above.