

Practice Exam Problems

1. Prove that there exists a real number r such that $r^2 = 2$
2. Prove that the real numbers are uncountable
3. Give the definition of a limit of a sequence.
4. State the nested interval theorem.
5. Give the definition of a supremum of a set and state the completeness axiom for the real numbers.
6. Prove that if $0 < b < 1$ then $\lim_{n \rightarrow \infty} b^n = 0$
7. Show that if x_n for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} x_n = x$ then $\lim_{n \rightarrow \infty} \sqrt{x_n} = \sqrt{x}$
8. Prove that any convergent sequence is bounded.
9. State the monotone convergence theorem for sequences
10. Show that if $\lim_{n \rightarrow \infty} z_n = z$ and $z_n > 0$ and $z > 0$ then $\lim_{n \rightarrow \infty} \frac{1}{z_n} = \frac{1}{z}$
11. Let $x_1 = 8$ and $x_{n+1} = \frac{1}{2}x_n + 2$. Show that (x_n) is monotone and find the limit $\lim_{n \rightarrow \infty} x_n$
12. Prove that if x and y are real numbers with $x < y$ then there exists a rational number r with $x < r < y$.
13. Prove that $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = 0$
14. Prove that $\lim_{n \rightarrow \infty} \frac{\sin(1/n)}{n} = 0$