Math 3351, Test # 1, Name _____

1. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Find the **Reduced** Row Echelon Form of $\boldsymbol{A}$

(b) Find the Rank($\boldsymbol{A}$) = _____

(c) Find a Basis for the Row Space of $\boldsymbol{A}$.

2. Given the two systems: (a) $\begin{cases} x - y &= 5 \\ 2x + 3y &= 0 \end{cases}$; (b) $\begin{cases} x & -y & & = 1 \\ x & & +z & = 1 \end{cases}$

(a) (i) Write in matrix form $\boldsymbol{Ax} = \boldsymbol{b}$ and write $[\boldsymbol{A}|\boldsymbol{b}]$; (ii) Determine whether each system is consistent or inconsistent; (iii) Decide if $\boldsymbol{b}$ is in the range space of $\boldsymbol{A}$; (iv) If the system is consistent use row reduction to find all solutions.

(b) (i) Write in matrix form $\boldsymbol{Ax} = \boldsymbol{b}$ and write $[\boldsymbol{A}|\boldsymbol{b}]$; (ii) Determine whether each system is consistent or inconsistent; (iii) Decide if $\boldsymbol{b}$ is in the range space of $\boldsymbol{A}$; (iv) If the system is consistent use row reduction to find all solutions.

3. Given the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

(a) Find the determinant of $\boldsymbol{A}$.

(b) Find the inverse of $\boldsymbol{A}$.

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4. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

5. Solve the Initial Value Problem $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ with $\begin{bmatrix} x \\ y \end{bmatrix}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors for $\boldsymbol{A}$.

(b) Find the general solution.

(c) Solve the initial value problem.
