

1. Determine which phrase applies to the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n+1}}{n^2 + n - 3}$
- (a) *converges absolutely* (b) *diverges*
- (c) *has an infinite radius of convergence* (d) *converges conditionally*
- (e) *is not alternating*

2. Determine which of the following statements are *true*.

(I) If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

(II) The Ratio Test cannot be used to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

(III) If $a_n > 0$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

- (a) (I) and (II) (b) None (c) (II) and (III)
- (d) (I) and (III) (e) (I), (II), and (III)

3. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$.

- (a) $(-1, 5]$ (b) $[-1, 1]$ (c) $[1, 3]$ (d) $[-1, 5)$ (e) $(1, 3)$

4. Use power series to compute $\lim_{x \rightarrow 0} \frac{\ln(1-x^6) + x^6}{\sin(x^4) - x^4}$.

- (a) $1/2$ (b) 0 (c) 1 (d) 3 (e) $1/6$

5. A sequence is defined recursively by $a_1 = 1$ and $a_{n+1} = \frac{1}{4}(a_n + 5)$ for $n \geq 1$. Assuming the sequence is increasing and bounded above, find the limit $\lim_{n \rightarrow \infty} a_n$.

- (a) $7/4$ (b) 2 (c) $5/3$ (d) $3/2$ (e) $9/5$

6. Derive the Taylor series for $f(x) = \ln(x)$ centered at $a = 1$.

- (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n$ (c) $-\sum_{n=1}^{\infty} \frac{1}{n} (x-1)^n$
- (d) $\sum_{n=1}^{\infty} (-1)^{n+1} (x-1)^n$ (e) $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

7. Find a power series representation of $f(x) = \frac{x}{1+2x^2}$.

- (a) $\sum_{n=0}^{\infty} (-2)^n x^{2n+1}$ (b) $\sum_{n=0}^{\infty} \frac{1}{2^n} x^{2n+1}$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n$
- (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2} x^{n+1}$ (e) $\sum_{n=0}^{\infty} 2^n x^{2n}$

8. Use the Binomial Series to expand $\frac{1}{(1-x^2)^{1/3}}$ as a power series.

- (a) $1 + x^{2/3} + x^{4/3} + x^2 + \dots$ (b) $1 - \frac{1}{3}x + \frac{2}{3}x^2 - \frac{2}{9}x^3 + \dots$
- (c) $1 + \frac{1}{3}x^2 + \frac{2}{9}x^4 + \frac{14}{81}x^6 + \dots$ (d) $1 + \frac{1}{3}x^2 + \frac{1}{9}x^4 + \frac{7}{27}x^6 + \dots$
- (e) $1 - \frac{1}{3}x^2 + \frac{1}{9}x^4 + \frac{7}{81}x^6 + \dots$

9. Estimate the error of approximating the series $\sum_{n=2}^{\infty} \frac{n}{(n^2-1)^2}$ by $\frac{2}{3^2} + \frac{3}{8^2} + \frac{4}{15^2}$.

- (a) $4/225$ (b) $3/16$ (c) $1/30$ (d) $1/48$ (e) $5/288$

10. Find the sum of the series $\sum_{n=0}^{\infty} \frac{2}{5^{n+1}}$.

- (a) 2 (b) $5/4$ (c) $1/2$ (d) $2/5$ (e) $5/2$

11. Determine which of the following statements applies to the series $\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{n^n}$.

- (a) Diverges by the root test.
(b) Converges by the alternating series test.
(c) Diverges by the alternating series test.
(d) The root and alternating series tests are inconclusive.
(e) Converges by the root test.

12. Determine which *one* of the following series converges.

- (a) $\sum_{n=1}^{\infty} \frac{n-1}{n^2+1}$ (b) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$ (c) $\sum_{n=1}^{\infty} (-1)^n$
(d) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ (e) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$