Errata for Manifolds and Differential Geometry

Jeffrey M. Lee

DEPARTMENT OF MATHEMATICS AND STATISTICS, TEXAS TECH UNIVERSITY, LUBBOCK, TEXAS, 79409

Current address: Department of Mathematics and Statistics, Texas Tech University, Lubbock, Texas, 79409

E-mail address: `jeffrey.lee@ttu.edu`

0.1. Main Errata

Chapter 1

(1) Page 3. Near the end of the page "...we have a second derivative $D^2f(a) : \mathbb{R}^n \to L(\mathbb{R}^n, \mathbb{R}^m)$" should read
"...we have a second derivative $D^2f(a) : \mathbb{R}^n \to L(\mathbb{R}^n, L(\mathbb{R}^n, \mathbb{R}^m))$"

(2) . Page 4. The is a missing exponent of 2 in the partial derivatives in the display at the top of the page. It should (of course) read as

$$u^i = \sum_{j,k} \frac{\partial^2 f^i}{\partial x^j \partial x^k}(a) v^j w^k.$$

(3) Page 19, forth line from the top at the beginning of Notation 1.44:

$$(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1}$$

should be changed to

$$(x_1, \ldots, x_{n+1}) \in \mathbb{R}^{n+1} \backslash \{0\}$$

(4) Page 23. 6th line from the top. The expression $\mathtt{y} \circ f \circ \mathtt{x}|_{\alpha}^{-1}$ should be changed to $\mathtt{y}_\beta \circ f \circ \mathtt{x}_\alpha|_U^{-1}$

(5) Page 28. In the display at the bottom of the page, change the x in the exponential to a t.

Chapter 2

(1) Page 61. The curve defined in the 6th line from the bottom of the page is missing a t and should read as follows:

$$c_i(t) := \mathtt{x}^{-1}(\mathtt{x}(p) + t\mathbf{e}_i),$$

(2) Page 68. In Definition 2.19: "...so a tangent vector in T_qM" should read "...so a tangent vector in T_qN".

(3) Page 73. At the end of the second display, (M_2, p) should be changed to (M_2, q).

(4) Page 109. In the proof of Theorem 2.113, the first display should read

$$X_1(p), \ldots, X_k(p), \left.\frac{\partial}{\partial y^{k+1}}\right|_p, \ldots, \left.\frac{\partial}{\partial y^n}\right|_p$$

(The initially introduced coordinates are $y^1, \ldots., y^n$.)

(5) On Page 115, in the second set of displayed equations there is both a spurious q and a spurious p (although, the action of the derivations

in question are indeed happening at the respective points: The display should read as follows:

$$\begin{aligned}(\phi^* df)|_p\, v &= df|_q\,(T_p\phi \cdot v)\\ &= ((T_p\phi \cdot v)\, f)\\ &= v\,(f \circ \phi) = d(\phi^* f)|_p\, v.\end{aligned}$$

(6) Page 124. Problem (18). The second to last line of part (b) we should find $T_0 R^n = \Delta_\infty \cong \left(\mathfrak{m}_\infty/\mathfrak{m}_\infty^2\right)^*$.

Chapter 3

(1) Page 167, Equation 4.3.

Equation 4.3 is missing a square root. It should read

$$L(c) = \int_a^b \left(\sum_{i,j=1}^{n-1} g_{ij}(c(t)) \frac{dc^i}{dt}\frac{dc^j}{dt}\right)^{1/2} dt,$$

where $c^i(t) := u^i \circ c$.

(2) The last displayed set of equations on page 175 is missing two square roots in the integrands and should read as follows:

$$\begin{aligned}L(\gamma) = \int_a^b \|\dot{\gamma}(t)\|\; dt &= \int_a^b \left\langle \sum \frac{du^i}{dt}\frac{\partial \mathbf{x}}{\partial u^i}, \sum \frac{du^j}{dt}\frac{\partial \mathbf{x}}{\partial u^j} \right\rangle^{1/2} dt\\ &= \int_a^b \left(\sum_{i,j}^{n-1} g_{ij}(u(t)) \frac{du^i}{dt}\frac{du^j}{dt}\right)^{1/2} dt,\end{aligned}$$

Chapter 5

(1) Page 212. In the first displayed equation in the proof of Theorem 5.7.2, the time derivative should be taken at an arbitrary time t and so $\frac{d}{dt}\big|_{t=0}\, \bar{c}(t)$ should be changed to $\frac{d}{dt}\bar{c}(t)$.

(2) Page 218. The latter part of the third sentence in the proof of Theorem 5.81 on page 218 should read "is a relatively closed set in U".

Chapter 6

(1) Insert the following definition after the exercise on page 271:

If (E, π, M, V) is a vector bundle, then we also have the associated **dual bundle** $(E^*, \pi^*, M, \mathrm{V}^*)$. Here $E^* = \bigcup_{p\in M} E_p^*$. The vector bundle charts are obtained by duality on each fiber in the obvious way.

(2) Page 285. First line of Section 6.5. We need not assume the same typical fiber in the two bundles. Change to read as follows:

Let $\xi_1 := (E_1, \pi_1, M, \mathrm{V})$ and $\xi_2 := (E_2, \pi_2, M, \mathrm{W})$

Chapter 9

(1) Page 395. All integrations in the first set of displayed equations should be over the entire space $\mathbb{R}^n$. (In an earlier version I had put the support in the left half space but after changing my mind I forgot to modify these integrals.)

(2) Page 395. In "Case 2" right *before* the line the begins "If $j = 1$", the variable u^1 ranges from $-\infty$ to 0 and so "$= \int_{\mathbb{R}^{n-1}}$" should be replaced by "$= \int_{\mathbb{R}^{n-1}_{u^1\leq 0}}$".

(3) Page 395. In "Case 2" right *after* the line the begins "If $j = 1$", the displayed equation should read

$$\int_{\mathbb{R}^n_{u^1\leq 0}} d\omega_1 = \int_{\mathbb{R}^{n-1}} \left(\int_{-\infty}^{0} \frac{\partial f}{\partial u^1} du^1 \right) du^2 \cdots du^n$$

(4) Page 396. In the first line of the second set of displayed equations a U_α need to be changed to M. Also, we use the easily verified fact that $\int_M \omega = \int_U \omega$ when the support of ω is contained in an open set U. (We apply this to the $\rho_\alpha\omega$ in the proof.) The display should read:

$$\begin{aligned}\int_M d\omega &= \int_M \sum_\alpha d(\rho_\alpha\omega) = \sum_\alpha \int_{U_\alpha} d(\rho_\alpha\omega) \\ &= \sum_\alpha \int_{\mathtt{x}_\alpha(U_\alpha)} (\mathtt{x}_\alpha^{-1})^* d(\rho_\alpha\omega) = \sum_\alpha \int_{\mathtt{x}_\alpha(U_\alpha)} d((\mathtt{x}_\alpha^{-1})^* \rho_\alpha\omega) \\ &= \sum_\alpha \int_{\partial\{\mathtt{x}_\alpha(U_\alpha)\}} ((\mathtt{x}_\alpha^{-1})^* \rho_\alpha\omega) = \sum_\alpha \int_{\partial U_\alpha} \rho_\alpha\omega = \int_{\partial M} \omega,\end{aligned}$$

(5) Page 396-397. Starting at the bottom of 396 and continuing onto 397, all references to the interval $(-\pi, \pi)$ should obviously be changed to $(0, \pi)$. The integrals at the top of page 397 should be changed accordingly.

(6) Page 399. The last line before the final display should read "where $(\rho X^k)_i := d(\rho X^k)(E_i)$ for all k, i."

Chapter 10

(1) On page 444 immediately before Definition 10.3, the following should be inserted: Recall that if a sequence of module homomorphisms, $\cdots \to A_{k-1} \overset{f_k}{\to} A^k \overset{f_{k+1}}{\to} A_{k+1} \to \cdots$ has the property that $\mathrm{Ker}(f_{k+1}) = \mathrm{Im}(f_k)$, then we say that the sequence is exact at A_k and the sequence is said to be **exact** if it is exact at A_k for all k (or all k such that the kernel an image exist).

(2) Page 444. In the middle of the page, the sentence that defines a chain map should state that the map is linear. Indeed, in this section it is likely that all maps between modules or vector spaces are linear by default.

(3) Page 444. The last line before definition 10.5 should read "$x \in Z^k(A)$". The following should be added: Note that if $x - x' = dy$ then $f(x) - f(x') = f(dy) = d(f(y)$ so that $f(x) \sim f(x')$.

(4) Page 451. In the second and third lines of the first display of "Case I", the factor $(-1)^k$ should be omitted (in both lines). This factor does appear in the forth line.

(5) Page 451. In the second to last display there is a spurious "*". Replace $(f \circ s_a \circ \pi^*)\, \pi^* \alpha$ by $(f \circ s_a \circ \pi)\, \pi^* \alpha$.

(6) Page 457. Top of the page. Here the author accidentally changed notation. The map denoted $\iota_0^* + \iota_1^*$ should just be denoted by j^*. Also, two instances of $\operatorname{Ker} d^*$ should be replaced by $\operatorname{Im} d^*$. Thus the last part of the proof should read:

...for a given q

$$\longrightarrow H^q(M_k \cap U_{k+1}) \xrightarrow{d^*} H^q(M_{k+1}) \xrightarrow{j^*} H^{q+1}(M_k) \oplus H^{q+1}(U_{k+1}) \longrightarrow,$$

which gives the exact sequence

$$0 \longrightarrow \operatorname{Im} d^* \longrightarrow H^q(M_{k+1}) \xrightarrow{j^*} \operatorname{Im}(j^*) \longrightarrow 0.$$

Since $H^q(M_k \cap U_{k+1})$ and $H^{q+1}(M_k) \oplus H^{q+1}(U_{k+1})$ are finite-dimensional by hypothesis, the same is true of $\operatorname{Im} d^*$ and $\operatorname{Im}(j^*)$. It follows that $H^q(M_{k+1})$ is finite-dimensional and the induction is complete.

Chapter 12

(1) Page 502. Line 14 from the top. We should find the following: "Thus we get a well-defined map $\nabla : TM \times \Gamma(M, E) \to E$ such that..."

(2) Page 502. That $\nabla_v s \in E_{\pi(v)}$ should be made clear. The map respects fibers. We could adjust the first required property to read

(i') $\nabla_{av}(s) = a \nabla_v s \in E_{\pi(v)}$ for all $a \in \mathbb{F}$, $v \in TM$ and $s \in \Gamma(M, E)$;

(3) Page 505. 13 lines from the bottom. Correct by including the base point p:

$$L(T_pM, E_p) \cong E_p \otimes T_p^* M$$

(4) Page 508. The correct form for a covariant derivative along a map is supposed to be $\nabla^f : TN \times \Gamma_f(E) \to E$. **Teachable moment:** The reason that the author made this error is because of a vacillation between to ultimately equivalent ways of defining a covariant

derivative. We either have

$$\nabla^f : TN \times \Gamma_f(E) \to E$$

or

$$\nabla^f : \mathfrak{X}(N) \times \Gamma_f(E) \to \Gamma_f(E).$$

The latter is derived from the former according to the prescription that for $U \in \mathfrak{X}(N)$ we let $\nabla^f_U \sigma$ be defined by $p \mapsto \nabla^f_{U(p)}\sigma$ as explained in the text.

(5) Page 510. In the proof of the lemma the N should be E_p: The first sentence of the proof should start as follows: $\pi \circ \imath \circ \gamma$ is constant for each smooth curve γ in E_p,...

(6) Page 511. In the last line of Exercise 12.10 change $M \times E$ to $N \times E$.

(7) Page 511. Line 6 from the bottom. Change "complex vector bundle" to "complex vector space".

Appendices

(1) Page 639, Definition A.4,.
"A morphism that is both a monomorphism and an epimorphism is called an isomorphism."
should be changed to
"A morphism that is both a monomorphism and an epimorphism is called a **bimorphism**. A morphism is called an **isomorphism** if it has both a right and left inverse."

(2) Page 648. In the constant rank theorem a q needs to be changed to a 0. We should read "...there are local diffeomorphisms $g_1 : (\mathbb{R}^n, p) \to (\mathbb{R}^n, 0)$".