Math5333: Partial Differential Equations II
Section 5333-001, Spring 2026.

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◇ **Lectures time and place:** MWF 14:00-14:50 in classroom MATH 108.

◇ **Office hours.** Mondays from 3:00pm to 5:00pm. If that time does not fit your schedule, please send me an e-mail so we can better coordinate. I surely accommodate one-to-one meetings on Mondays and Fridays.

◇ **Textbook:** The official textbook is "Partial Differential Equations", 2nd Edition (2010), authored by Lawrence C. Evans, published by the American Mathematical Society. Another excellent textbook is: "Partial Differential Equations in action", 3rd Edition (2016), by Sandro Salsa, published by Springer. The contents of the book by Salsa are almost the same as those by Evans. However, I find the book by Salsa (sometimes) easier to read. I will prepare my lectures using both books.

◇ **Contents of this class (PDE-II):** This class is meant to prepare graduate students to take their preliminary examination on Partial Differential Equations. The contents of the preliminary exam on PDEs are described in:

$$\text{https://www.depts.ttu.edu/math/graduate/resources/prelims/PDE/PDETopics.pdf}$$

While PDE-II may be taken in isolation, that is not advisable: PDE-I and PDE-II are meant to be taken as a year-long sequence.

PDE-II will focus on the so-called "weak framework" or "variational approach" to the solution of Partial Differential Equations. In essence: PDE problems not are assumed to have classical solutions. Boundary value problems and initial value problems are recast in abstract functional form: "bilinear form = functional in the dual space". This reformulation allows us to use heavy machinery from functional analysis and kill the problems in one shot: well-posedness and uniqueness are proved straightforwardly using Lax-Milgram theorem. However, proving that such reformulation is well-posed requires developing a proper functional setting. In this context, the notion of "norm equivalence" plays a pivotal role. Some fundamental tools used to prove norm equivalence are Sobolev, Poincare and Trace inequalities. We will spend 40% of the semester developing such functional tools.

Roughly speaking, the class will cover the content of chapters 5, 6, and 7 of Evans' book. Or equivalently: chapters 6, 7, 8, and 10 from Salsa's book. Both books complement each other. The book of Evans is somewhat more complete, but the book of Salsa has some particularly concise and simple proofs (as well as examples) that are worth taking a look.

Note: PDE-II will NOT cover representation formulas, fundamental solutions, green's functions, classical/strong solutions, scalar conservation laws, methods of characteristics, etc. Those are the

topics from Chapters 1, 2 and 3 of the book of Evans. We covered all those topics in PDE-I during Fall 2025. PDE-I will be offered again this Fall 2026 as well.

◇ **Learning Outcomes.** By the end of the class the student should have knowledge and skills about the following topics:

- *Rudiments of functional analysis:* Bilinear forms and bounded linear functionals, Abstract Variational Problems, Lax-Milgram theorem, Spectral Theory for Symmetric Bilinear Forms.
- *Function spaces:* Lipschitz, Holder and Sobolev spaces, Approximation by smooth function, Extension Operators, Trace Operators, Sobolev inequalities, Poincare-like inequalities, Morrey inequalities, Compact Embeddings (Rellich-Kondrachov theorem).
- *Variational formulation of elliptic problems:* Existence of Weak solutions, Regularity, Weak maximum principle, Eigenvalues and eigenfunctions.
- *Variational formulation of linear evolution equations:* Second order Parabolic equations, Second order Hyperbolic equations, Faedo-Galekin Method, Sobolev spaces involving time (Bochner-Spaces), Hyperbolic systems of first-order equations, Friedrich's systems, compactness of Bochner spaces (Aubin-Lions lemma), Semigroup theory.

◇ **Required background:** Strong background in multi-variable calculus is mandatory. PDE-II requires some knowledge of functional and real analysis. However, we will not develop functional or real analysis results: we will make use of them. We will review the required tools from functional and real analysis during the course of the class. You might have to fill-in some gaps (e.g. reading some notes on functional analysis on your own) but nothing too overwhelming.

Contrary to PDE-I, PDE-II is not an elementary class, in the sense that we will use somewhat advanced tools. However, once these tools are properly assimilated, solving problems will be easy.

◇ **Homework:** this class will have 5-6 homework assignments. You will be given 10-14 days to complete each homework assignment. Homework is the most important component of this class. It is your responsibility to do and latex/hand-write your own homework. Your homework is not merely for submission: it is a written record of things you have learned that you should use to study for the final exam. The goal of the homework is to teach you mathematical arguments and techniques that are <u>standard</u> and <u>widely used</u> in the scientific literature.

◇ **Exams:** there will be no midterms.

◇ **Final Exam:** There are two options here:

- *Option #1: oral final exam.* Assuming that we have a moderate number of students, I will take oral (in-person) final exams from April 30th to May 12th. We will use an online scheduler where you will pick what day and time you will take your final: first-come firt-serve basis. You will have to show-up to my office with your all your homework (either originals, or printed copies, or photocopy of your hand-written homework). I will pick 2-3 problems from the homework, I might introduce some modifications/variations to the problems, and you will have to either solve the problems on the blackboard, or explain to me what is a proper solution strategy (if the actual solution is deemed to lengthy).

- *Option #2: take home final exam.* If the total volume of students is too big to handle, you will have a take-home exam (open book, open notes), sometime between May 7th to May 12th. The exam will consists of 4-5 problems and have 4 hours in duration. You will have to scan (photograph) your answers and send the corresponding documents via e-mail. You are allowed to hand-write your answers in your favorite note-taking software on your tablet.

Premature booking of flights has been a source of conflict with finals in the past. Please, avoid booking flights anytime before May 12th.

⋄ **Grade:** 50% of your grade will come from your homework, 50% will come from your final exam.

⋄ **Important dates.** You can find the (full) official TTU's academic calendar at

`https://www.depts.ttu.edu/officialpublications/calendar/25-26_onepage_calendar.pdf`

Some important TTU dates I extracted from the official calendar:

- Classes Begin: January 14th.
- MLK day: January 19.
- Spring break: March 14th-22nd.
- No classes: April 6th.
- No Exams Except Makeup or Scheduled Lab Exams: April 30th - May 6th.
- Last Day of Classes: May 5th.
- Individual Study day: May 6th (No lecture).
- Final Examinations: May 7th to May 12th.
- Semester Ends: May 12th.

Warning! Technically speaking, according to the academic calendar, your final is on Tuesday, May 12th. Therefore, make proper arrangements, and by all means avoid booking flights before May 12th.

Note: these dates are provided for your convenience. This syllabus should NOT be used as your primary academic calendar reference. Always refer to the official academic calendar.

⋄ **Grade posting:** in order to comply with state regulations and preserve your right of privacy, I will NOT communicate grades by e-mail or phone. I will strictly communicate and post grades using RaiderCanvas or in person during office hours.

◇ **Texas Tech Operating Policies and Procedures.** The complete policies are available at

http://www.depts.ttu.edu/opmanual/

The operaring policies are numerous but here are three that are particularly important:

- Academic Honesty (OP 34.12): It is the aim of the faculty of Texas Tech University to foster a spirit of complete honesty and high standard of integrity. The attempt of students to present as their own any work not honestly performed is regarded by the faculty and administration as a most serious offense and renders the offenders liable to serious consequences, possibly suspension. "Scholastic dishonesty" includes, but it not limited to, cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, and any act designed to give unfair academic advantage to the student (such as, but not limited to, submission of essentially the same written assignment for two courses without the prior permission of the instructor) or the attempt to commit such an act.

- ADA Accommodation (OP 34.22): Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note that instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, you may contact the Student Disability Services office at 335 West Hall or 806-742-2405.

- Religious Holy Day Observance (OP 34.19): "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20. A student who intends to observe a religious holy day should make that intention known to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence. A student who is excused may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.