Syllabus: MATH 5344, Fall 2024

Official title: Topics Numerical Analysis II

Numerical Analysis of PDEs, Part II: time-dependent problems.

General information

- ◇ ***Instructor:*** Ignacio Tomas
- ◇ ***Office:*** MATH221
- ◇ ***E-mail:*** `igtomas@ttu.edu`
- ◇ ***Webpage:*** `https://www.math.ttu.edu/~igtomas/Teaching`
- ◇ ***When and where:*** TBA.
- ◇ ***Homework:*** to be sent by e-mail.

Bibliography

There is no *main* textbook. The instructor will develop lecture using the following books:

The lectures will be prepared using selected chapters/material from:

- ◇ S. Larrson and V. Thomee, Partial Differential Equations with Numerical Methods, 2005.
- ◇ J-L. Guermond and A. Ern, Theory and Practice of Finite Elements, 2004.
- ◇ E. Godlewski and P-A. Raviart, Numerical Approximation of Hyperbolic Systems of Conservation Laws, 1996.
- ◇ E. Toro, Riemann solvers and Numerical Methods for Fluid Dynamics, 2009.
- ◇ A. Ern and D. DiPietro, Mathematical Aspects of Discontinuous Galerkin Methods, 2011.
- ◇ A. Quarteroni and A. Valli, Numerical Approximation of Partial Differential Equations, 2008.
- ◇ K. Atkinson, W. Han, Theoretical Numerical Analysis, 2009.
- ◇ Lawrence C. Evans, Partial Differential Equations, 2022.
- ◇ Sandro Salsa, Partial Differential Equations in Action, 2022.

Contents of the class

The official title of this class is "Topics Numerical Analysis I". However, a much more accurate title for this class would be "Numerical Analysis of PDEs, Part II: Time Dependent Problems".

- ◇ *Parabolic problems in Hilbert space framework:* Heat equation; Navier-Stokes equation; Bochner norms; Aubin-Lions lemma; energy estimates and dual-norm estimates.
- ◇ *Linear hyperbolic problems in the Hilbert space framework:* Linear transport; Acoustic Wave equation; Maxwell's system; Friedrich's systems.
- ◇ *Basic methods for scalar hyperbolic problems:* Centered finite differences; Upwind finite differences; discontinuous Galerkin discretization; jump penalization.
- ◇ *Time integration for parabolic problems:* Forward-Euler, Backward-Euler and Crank-Nicolson method; Diagonally implicit Runge-Kutta methods; continuous and discrete Gronwall's lemma.
- ◇ *Non-linear hyperbolic problems:* non-linear scalar conservation laws; hyperbolicity and projected Riemann problems. Vanishing viscosity principle: entropy inequalities, maximum and minimum, principles. Nonlinear systems of conservation laws: invariant sets and entropy inequalities.
- ◇ *Numerical methods for non-linear hyperbolic problems:*
  - — First-order methods in 1d: Godunov method, staggered Lax method, artificial viscosity methods.

- — Multidimensional methods: cell-based discontinuous finite elements, stencil based-methods (graph-based methods)
- — Preservation of pointwise stability: slope-limiting and flux-limiting
- — SSP time integration

◇ *Practical aspects (deal.ii):* Introduction to C++, compiler and pre-processor, sparsity graphs, Cell-loops, gather and scatter operations, iterative solvers and preconditioners.

Course evaluation

This class will have NO midterms. The course will have a total of 8 homework assignments. The homework has to be turned in by its deadline, which will be about two weeks after its assignment. The homework will be composed of 30% PDE-theory problem (proof-based), 50% Numerical Analysis theory (proof-based), and %20 practical assignments requiring coding with Matlab or Python. Final examination is oral: it will consist of a review, extension and/or modification of the problems considered in the homework and their discussion on the blackboard. You are encouraged to discuss homework problems during: plase avoid getting stuck on your own. The letter-grade brackets are:

$$\mathrm{F} : [0, 60)\,,\ \mathrm{D} : [60, 70)\,,\ \mathrm{C} : [70, 80)\,,\ \mathrm{B} : [80, 90)\,,\ \mathrm{A} : [90, 100]\,.$$

Texas Tech Operating Policies and Procedures

The complete policies are available at

`http://www.depts.ttu.edu/opmanual/`

The operating policies are numerous but here are three that are particularly important:

◇ Academic Honesty (OP 34.12): It is the aim of the faculty of Texas Tech University to foster a spirit of complete honesty and high standard of integrity. The attempt of students to present as their own any work not honestly performed is regarded by the faculty and administration as a most serious offense and renders the offenders liable to serious consequences, possibly suspension. "Scholastic dishonesty" includes, but it not limited to, cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, and any act designed to give unfair academic advantage to the student (such as, but not limited to, submission of essentially the same written assignment for two courses without the prior permission of the instructor) or the attempt to commit such an act.

◇ ADA Accommodation (OP 34.22): Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note that instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, you may contact the Student Disability Services office at 335 West Hall or 806-742-2405.

◇ Religious Holy Day Observance (OP 34.19): "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20. A student who intends to observe a religious holy day should make that intention known to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence. A student who is excused may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.