# Syllabus: MATH 5345, Spring 2024
## *Official title: Topics Numerical Analysis II*

Numerical Analysis of PDEs, Part I: stationary problems.

---

### General information

- ***Instructor:*** Ignacio Tomas
- ***Office:*** MATH221
- ***E-mail:*** `igtomas@ttu.edu`
- ***Webpage:*** `https://www.math.ttu.edu/~igtomas/Teaching`
- ***When and where:*** we meet for lectures MWF 11:00 AM-11:50 AM in HOLDEN 233. However, if we manage to get a room in the Math building we will change the room.
- ***Homework:*** to be sent by e-mail.
- ***Office hours:*** Tuesdays and Thursdays from 10:00AM to noon. Alternatively, by appointment.

---

### Contents of the class

The official title of this class is “Topics Numerical Analysis II”. However, a much more accurate title for this class would be “Numerical Analysis of Partial Differential Equations, Part I: Stationary problems”. More precisely, this course will cover elliptic and stationary hyperbolic problems (transport) during Spring 2024. Part II will make an emphasis on time-dependent parabolic and hyperbolic problems during Fall 2024. These two courses do NOT intend to be a replacement of a graduate course sequence on PDEs. However, we will cover the rudiments of classical/strong solutions as well as modern PDE theory (e.g. weak solutions in the context of Sobolev spaces) at the level of the book of L. Evans.
Part I will cover the following:

- *Rudiments of Functional Analysis:* Hilbert and Banach spaces, multi-d integration by parts, weakly differentiable functions, Sobolev Spaces, Lipschitz and Holder spaces, Gagliardo-Niremberg-Sobolev inequalities, Morrey inequality, Rellich-Kondrachov theorem, Poincare’s inequalities, Friedrichs’s inequalities, Trace inequalities.
- *Interpolation:* polynomial interpolation, simplicial and tensor product finite elements, scaling arguments and inverse inequalities, Deny-Lions lemma, Bramble-Hilbert lemma, interpolation operators for non-smooth functions.
- *Galerkin methods:*
  - *Linear problems in Banach spaces:* Banach-Nečas-Babuška theorem, coercivity and Lax-Milgram theorem, Cea’s and related quasi-optimality lemmas, Aubin-Nitsche method, abstract saddle-point problems.
  - *Scalar problems:* Elliptic PDEs, Conforming approximation, Nitsche-like boundary conditions, Interior penalty approximation.
  - *Mixed and saddle-point problems:* Mixed Laplacian, Stokes, Biharmonic mixed-formulation, Eddy-Currents, Stable-pairs, Fortin’s lemma, Linear algebra of mixed problems.
  - *First-order stationary PDEs:* Galerkin Least-Squares, Discontinuous Galerkin.
- *Non-Galerkin methods and properties:* Finite difference methods, finite volume methods, Poisson problem in 1d, M-matrices, discrete maximum and mimimum principles, and L-infinity estimates using barrier functions. If time allows, we will explore a little bit of finite volume methods on Voronoi meshes for Elliptic PDEs.

---

## Bibliography

There is no *main* textbook. The instructor will develop the lectures using the following books:

- ◇ S. Larrson and V. Thomee, Partial Differential Equations with Numerical Methods, 2005.
- ◇ J-L. Guermond and A. Ern, Theory and Practice of Finite Elements, 2004.
- ◇ A. Ern and D. DiPietro, Mathematical Aspects of Discontinuous Galerkin Methods, 2011.
- ◇ Eymard, Gallouët, Herbin, Finite Volume Methods, Handbook of Numerical Analysis, 2000.
- ◇ Grossman, Roos, Stynes, Numerical Treatment of Partial Differential Equations, 2007.
- ◇ K. Atkinson, W. Han, Theoretical Numerical Analysis, 2009.
- ◇ Lawrence C. Evans, Partial Differential Equations, 2022.
- ◇ Sandro Salsa, Partial Differential Equations in Action, 2022.

In particular, we will follow: Larrson-Thomee Chapters 2-5 for the first 2-4 weeks in order to make things accessible. Then, we will switch to Ern-Guermond chapters 1-4 to focus on variational methods. We will use Ern-DiPietro for the last 3 weeks of the semester in order to study Interior Penalty methods for Elliptic problems and Discontinuous Galerkin methods for stationary first-order PDEs. If time allows we will discuss Finite Volume Methods for Elliptic PDEs on Voronoi meshes using the books of Eymard and Grossman-Roos. Various tools from Functional Analysis and Sobolev spaces will be picked as needed, when needed, from the books of Atkinson, Evans and Salsa.

---

## Expected learning outcomes

By the end of the course the student should be able to develop rigorous stability analysis of numerical methods used to solve linear second-order Elliptic PDEs and stationary first-order PDEs (transport). The student should have acquired significant competence in the use of standard analysis tools such Galerkin orthogonality, inf-sup stability, quasi-optimality lemmas, stable-pairs for mixed problems, interpolation estimates, and error estimates. Beyond the scope of Galekin methods, the student should also know the basics about non-variational properties: discrete maximum and minimum principles, L-infinity estimates using barrier-function techniques, and M-matrices. Equipped with such solid mathematical background, a student should be able to engage in independent reading and research of linear and nonlinear Elliptic PDEs as well as transport problems.

---

## Course evaluation

This class will have NO midterms. The course will have a total of 8 homework assignments. The homework has to be turned in by its deadline, which will be about two weeks after its assignment. The homework will be composed of approximately: 30% PDE-theory problem (proof-based), 60% Numerical Analysis theory (proof-based), and %10 practical assignments requiring coding with Matlab or Python. Final examination is oral: it will consist of a review, extension and/or modification of the problems considered in the homework and their discussion on the blackboard. You are encouraged to discuss homework problems with the instructor and with your peers: please avoid getting stuck on your own. The letter-grade brackets are: $\text{F} : [0, 60)\,,\ \text{D} : [60, 70)\,,\ \text{C} : [70, 80)\,,\ \text{B} : [80, 90)\,,\ \text{A} : [90, 100]\,.$

---

## From the academic calendar Spring 2024

Important dates:

- ◇ Jan 10: classes begin
- ◇ April 30: last day of classes.
- ◇ May 2-7: final examinations.
- ◇ May 7: semester ends

Hollidays:

- ◇ January 15: MLK days.
- ◇ March 9–17: Spring vacation
- ◇ April 1: No Classes

---

## Texas Tech Operating Policies and Procedures

The complete policies are available at

`http://www.depts.ttu.edu/opmanual/`

The operating policies are numerous but here are three that are particularly important:

- ◇ Academic Honesty (OP 34.12): It is the aim of the faculty of Texas Tech University to foster a spirit of complete honesty and high standard of integrity. The attempt of students to present as their own any work not honestly performed is regarded by the faculty and administration as a most serious offense and renders the offenders liable to serious consequences, possibly suspension. "Scholastic dishonesty" includes, but it not limited to, cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, and any act designed to give unfair academic advantage to the student (such as, but not limited to, submission of essentially the same written assignment for two courses without the prior permission of the instructor) or the attempt to commit such an act.
- ◇ ADA Accommodation (OP 34.22): Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note that instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, you may contact the Student Disability Services office at 335 West Hall or 806-742-2405.
- ◇ Religious Holy Day Observance (OP 34.19): "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20. A student who intends to observe a religious holy day should make that intention known to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence. A student who is excused may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.